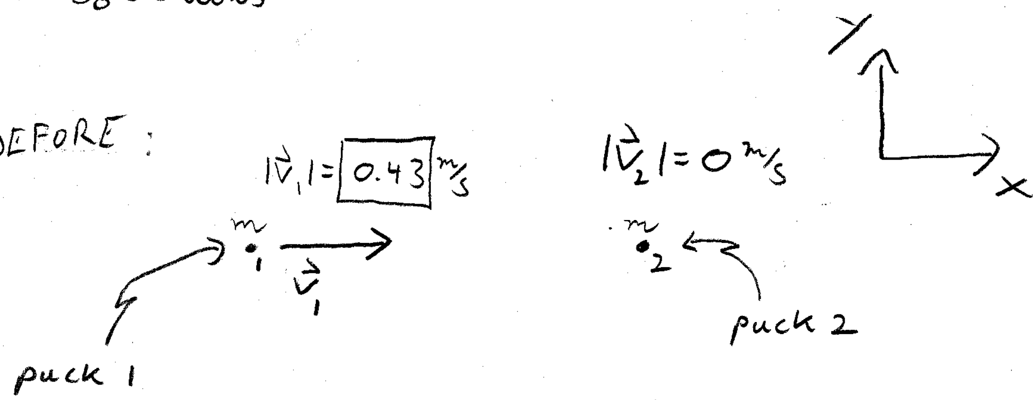
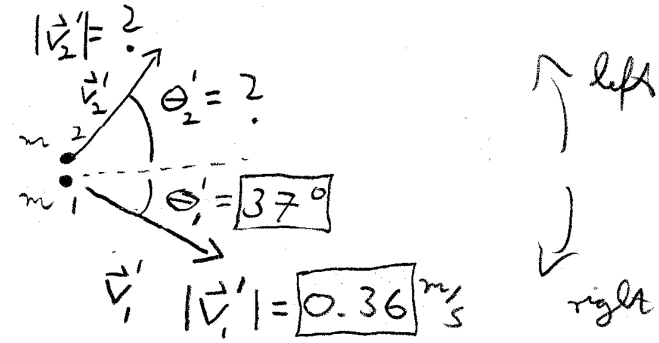
Hwk # 9 Solutions

① BEFORE:



AFTER:



Find $|\vec{v}_2'| = ?$

and $\theta_2' = ?$

– I used prime variables to denote final state after the collision and unprimed variables to denote initial state before collision.

Use momentum conservation:

$$\vec{P}_{tot} = \vec{P}_{tot}'$$

total momentum <u>before</u>

total momentum <u>after</u>

Total momentum is conserved as a <u>vector</u> ⇒

both x and y components of total momentum stay the same:

So

$$\boxed{p_{tot_x} = p'_{tot_x}}$$

total horizontal momentum before

total horizontal momentum after

and

$$\boxed{p_{tot_y} = p'_{tot_y}}$$

total vertical momentum before

total vertical momentum after

but $\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$ and $\vec{p}\,'_{tot} = \vec{p}_1\,' + \vec{p}_2\,'$

$\boxed{\text{BEFORE}}$

$\boxed{x}$

$$p_{tot_x} = m v_{1_x} + m v_{2_x}.$$

both pucks have same mass m!

$$\Rightarrow \quad p_{tot_x} = m \boxed{0.43} \text{ m/s} + 0$$

total x-momentum before

$\boxed{y}$

and

$$p_{tot_y} = m v_{1_y} + p_{2_y} \quad (= m_2 v_{2_y} = 0)$$

$$\Rightarrow \quad p_{tot_y} = 0 + 0$$

total y-momentum before

$\boxed{\text{AFTER}}$

unknown momentum of second puck in x

$\boxed{x}$ $$p'_{tot_x} = m v'_{1x} + p'_{2x}$$

$$\Rightarrow \quad p'_{tot_x} = m|\vec{v}_1'| \cos\theta_1' + p'_{2x}$$

$\boxed{y}$ $$p'_{tot_y} = m v'_{1y} + p'_{2y}$$

$$p'_{tot_y} = m|\vec{v}_1'| \sin\theta_1' + p'_{2y}$$

Use momentum conservation and solve for the unknown momentum components p'_{2x} and p'_{2y} of puck 2 after the collision and use the numbers everywhere

$\boxed{\text{BEFORE = AFTER}}$

$\boxed{x}$ $$p_{tot_x} = p'_{tot_x}$$

$$m\boxed{0.43\ \tfrac{m}{s}} = m\underbrace{\boxed{0.36\ \tfrac{m}{s}}}_{|\vec{v}_1'|}\cos\underbrace{37°}_{\theta_1'} + p'_{2x}$$

$$\Rightarrow \quad p'_{2x} = m\left(\boxed{0.43\ \tfrac{m}{s}} - \boxed{0.36\ \tfrac{m}{s}}\cos\boxed{37°}\right)$$

$\boxed{y}$ $$p_{tot_y} = p'_{tot_y}$$

$$0 = m \cdot \boxed{0.36\ \tfrac{m}{s}}\sin\boxed{37°} + p'_{2y}$$

$$\Rightarrow \quad p'_{2y} = -m \cdot \boxed{0.36\ \tfrac{m}{s}}\sin\boxed{37°}$$

So now we know:

$$p'_{2_x} = m\left(\boxed{0.43\ \tfrac{m}{s}} - \boxed{0.36\ \tfrac{m}{s}}\cos\boxed{37^\circ}\right)$$

$$p'_{2_y} = m\left(-\boxed{0.36\ \tfrac{m}{s}}\sin\boxed{37^\circ}\right)$$

But we want $|\vec{V}_2'|$, the final speed of puck 2, and θ_2', the direction of puck 2 after the collision.

We know

$$|\vec{V}_2'| = \sqrt{(V'_{2_x})^2 + (V'_{2_y})^2}$$

But $\vec{p} = m\vec{v}$, so we know V'_{2_x} and V'_{2_y}:

$\boxed{x}$ $$V'_{2_x} = \frac{p'_{2_x}}{m} \quad \left(\text{from } p'_{2_x} = m V'_{2_x}\right)$$

$$= \left(\boxed{0.43\ \tfrac{m}{s}} - \boxed{0.36\ \tfrac{m}{s}}\cos\boxed{37^\circ}\right) = 0.1425\ \tfrac{m}{s}$$

$\boxed{y}$ $$V'_{2_y} = \frac{p'_{2_y}}{m}$$

$$= -\boxed{0.36\ \tfrac{m}{s}}\sin\boxed{37^\circ} = -0.2167\ \tfrac{m}{s}$$

Then $$|\vec{V}_2'| = \sqrt{(V'_{2_x})^2 + (V'_{2_y})^2} = 0.259\ \tfrac{m}{s}$$

and $\theta_2' = \arctan\left(\dfrac{v_{2y}'}{v_{2x}'}\right)$

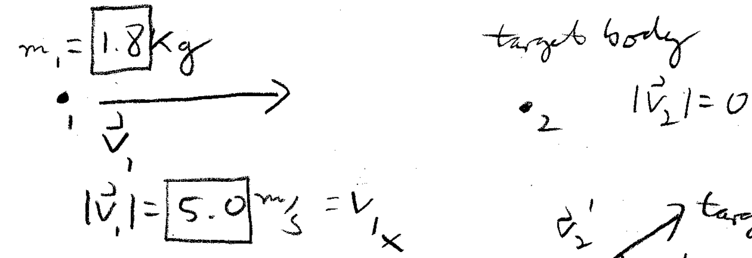
$= \arctan\left(\dfrac{-0.2167}{0.1425}\right)$

$= \arctan\left(-1.5207\right)$

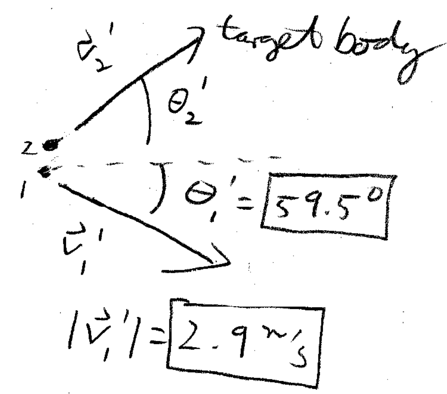
$\Rightarrow \quad \theta_2' = -56.67^\circ$ //

note: negative angle → deflection to the left!

② before:



after:



find $|\vec{P}_2'| = ?$

– use momentum conservation:

$\boxed{\text{BEFORE} = \text{AFTER}}$

$$P_{tot_x} = P'_{tot_x}$$

$$P_{tot_y} = P'_{tot_y}$$

v'_{1_x} in terms of what we <u>know</u>!

unknown momentum components!

$\boxed{\text{in } x}$: $m_1|\vec{v}_1| + 0 = \underbrace{m_1|\vec{v}_1'|\cos\theta_1'} + P'_{2_x}$

$\boxed{\text{in } y}$: $0 + 0 = m_1|v_1'|\sin\theta_1' + P'_{2_y}$

But we want $|\vec{P}_2'| = \sqrt{(P'_{2_x})^2 + (P'_{2_y})^2}$, so solve for P'_{2_x} and P'_{2_y} first:

$\boxed{\text{in } x}$ $P'_{2_x} = m_1|\vec{v}_1| - m_1|\vec{v}_1'|\cos\theta_1'$

$\boxed{\text{in } y}$ $P'_{2_y} = -m_1|\vec{v}_1'|\sin\theta_1'$

So with numbers I get

$$p'_{2x} = \boxed{1.8\,kg} \cdot \boxed{5.0\,m/s} - \boxed{1.8\,kg}\,\boxed{2.9\,m/s}\,\cos\boxed{59.5^\circ}$$

$$p'_{2y} = -\boxed{1.8\,kg} \cdot \boxed{2.9\,m/s}\,\sin\boxed{59.5^\circ}$$

then

$$p'_{2x} = \boxed{6.35065\,kg\,m/s}$$

$$p'_{2y} = \boxed{-4.4977\,kg\,m/s}$$

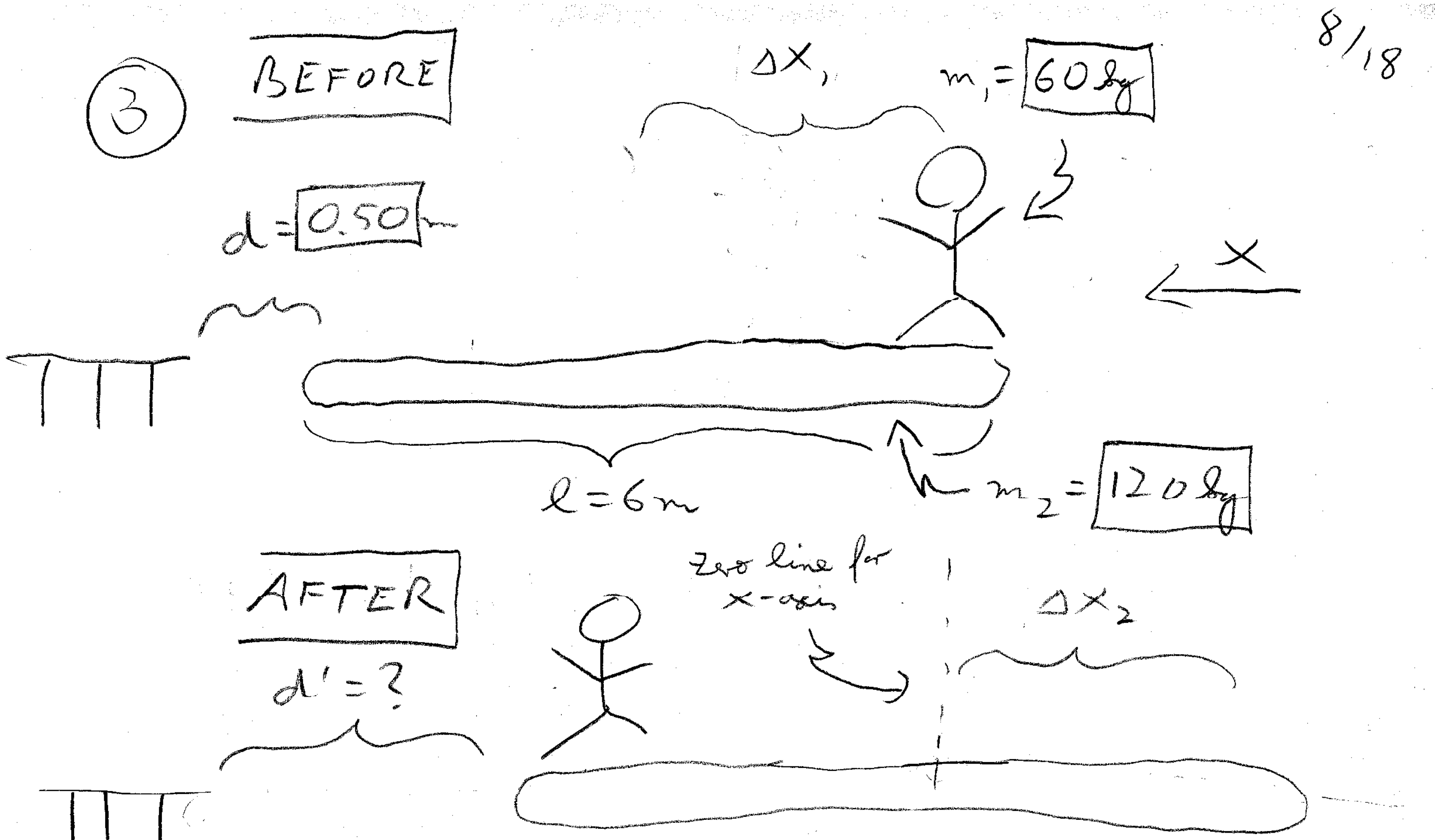
and thus

$$|\vec{p}\,'_2| = p'_2 = \sqrt{(p'_{2x})^2 + (p'_{2y})^2}$$

$$= \sqrt{\boxed{60.56}}\ kg\,m/s$$

$$p'_2 = \boxed{7.782}\ kg\,m/s$$

3



$$\left(|\Delta x_1| + |\Delta x_2| = 6\,m\right) \Rightarrow \boxed{\Delta x_1 + (-\Delta x_2) = 6\,m}$$

as Δx_2 is a negative displ.!

no outside forces $\Rightarrow$ momentum conservation

p = constant during the walk

$$\Rightarrow \quad p_{Before} = p_{during\ walk} = p_{After} = 0$$

it is zero as they both start from rest $\Rightarrow p_{Before} = 0$

$$\Rightarrow \quad p_{during\ walk} = 0$$

$$\Rightarrow \quad m_1 v_1 + m_2 v_2 = 0$$

$$m_1 v_1 = -m_2 v_2$$

$$\Rightarrow \quad v_2 = -\frac{m_1}{m_2} v_1$$

As the woman walks with $v_1 = \frac{\Delta x_1}{\Delta t}$

(she walks Δx_1 in some time Δt)

the raft moves at some speed $v_2 = \frac{\Delta x_2}{\Delta t}$ for the same time Δt $\Rightarrow$ plug this in:

$$\Rightarrow \quad -\frac{|\Delta x_2|}{\Delta t} = -\frac{m_1}{m_2}\frac{\Delta x_1}{\Delta t} = -\frac{\boxed{60\,kg}}{\boxed{120\,kg}}\frac{\Delta x_1}{\Delta t}$$

Δt cancels $\rightarrow$ no time dependence $\rightarrow$ does not matter how fast she walks.

$$\Rightarrow \quad \Delta x_2 = -\frac{m_1}{m_2}\Delta x_1 \quad , \text{we want } \Delta x_1 + (-\Delta x_2) = \boxed{6\,m}$$

as she needs to end up on other side

$$\Rightarrow \quad \Delta x_2 = -\frac{\boxed{60}}{\boxed{120}}(6\,m + \Delta x_2) \Rightarrow 1.5\,\Delta x_2 = -3\,m \Rightarrow \boxed{\Delta x_2 = -2\,m}$$

The raft moves $\boxed{2\,m}$ to the right in the time it takes the woman to walk across it!

$\Rightarrow$ It was 0.5 m from the pier, now it is $\boxed{2.5\,m}$ from the pier //

then with $\Delta x_2 = \boxed{-2\,m}$ from

$\Delta x_1 + (-\Delta x_2) = \boxed{6\,m}$ (see figure at beginning)

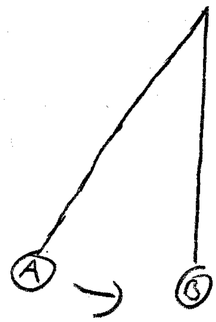
I get $\Delta x_1 = \boxed{6\,m} - \boxed{2\,m}$

$$\Delta x_1 = \boxed{4\,m}$$ ← But Δx_1 is the distance she walked relative to the pier (see figure)

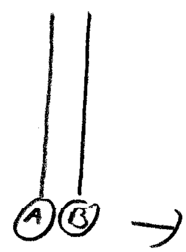
The woman walked a distance of 4 m relative to the pier!

(4) Before:

$m_A = \frac{1}{2} m_B$



After:



They stick together afterwards, B is initially at rest.

We look for $\frac{K'_{tot}}{K_A} = \frac{\text{Kinetic}E^{after}_{total}}{\text{Kinetic}E^{before}_{of\ A}}$

Before here means "just before" the impact, as problem says ⇒ They are made of soft clay and stick ⇒ no conservation of kinetic plus potential energy ⇒ some energy is lost to heat as they collide) ⇒ this is an inelastic collision because of the soft material. But conservation of momentum still holds in inelastic collisions.

use momentum conservation

$$P_{tot} = P_{tot}'$$

$\boxed{\text{BEFORE}}$

$$P_{tot} = P_A + P_B$$

$$= m_A V_A + m_B V_B \quad = 0!$$

$$= m_A V_A \qquad \text{use } m_A = \frac{1}{2} m_B$$

$$= \frac{1}{2} m_B V_A$$

$\boxed{\text{AFTER}}$

$$P_{tot}' = P_A' + P_B'$$

$$= m_A V_A' + m_B V_B'$$

But they stick $\Rightarrow$ move at same speed afterwards $\Rightarrow$

$$V_A' = V_B' = V'$$

$$P_{tot}' = m_A V' + m_B V'$$

$$= (m_A + m_B) V' \qquad (\text{use } m_A = \tfrac{1}{2} m_B)$$

$$= \frac{3}{2} m_B V'$$

$\boxed{\text{BEFORE} = \text{AFTER}}$

$$P_{tot} = P_{tot}'$$

use again $V_A' = V_B' = V'$ → after!

$$\Rightarrow \quad \frac{1}{2} m_B V_A = \frac{3}{2} m_B V' \quad ☺$$

We want the ratio $R = \dfrac{K_{tot}'}{K_A} = \dfrac{\frac{1}{2} m_A V_A'^2 + \frac{1}{2} m_B V_B'^2}{\frac{1}{2} m_A V_A^2}$

use $m_A = \frac{1}{2} m_B$ $\quad R = \dfrac{\left(\frac{1}{4} m_B + \frac{1}{2} m_B\right) V'^2}{\frac{1}{4} m_B V_A^2}$

$$R = \frac{3}{4} \cdot 4 \frac{V'^2}{V_A'^2}$$

$$R = 3 \cdot \frac{V'^2}{V_A^2}$$

get $\dfrac{V'^2}{V_A^2}$ from ☺ expression above:

$$\frac{V'}{V_A} = \frac{1}{3} \Rightarrow \frac{V'^2}{V_A^2} = \frac{1}{9}$$

$$\Rightarrow \quad R = 3 \cdot \frac{1}{9} = \frac{1}{3}$$

5 Use result from 4 :

$$\boxed{K'_{tot} = \frac{1}{3} K_A}$$

total kinetic E just after collision

kinetic energy of A just before collision

What is the total K of A at the instant just before collision? It is the amount of potential energy it lost in going from height h down to the height of B! It starts at height h from rest, so

$\Rightarrow \quad K_A = \Delta U = m_A g h$

Then use $K'_{tot} = \frac{1}{3} K_A = \frac{1}{3} m_A g h_i$ initial height of A

And now they both move up together with an initial kinetic energy of K'_{tot} until they reach the height where all of this K'_{tot} energy has become potential energy:

$\Delta E = 0$

$\Delta K + \Delta U = 0$, $K_{final} = 0$ on the way up, $U_{initial} = 0$, they start from the bottom

$U_{final} = K_{initial} = K_{tot}'$ ← they start with this on their way up!

$\downarrow$

$m_A g h_f + m_B g h_f = K_{tot}'$

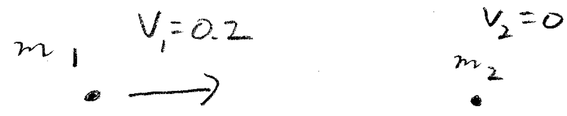
$\Rightarrow (m_A + m_B) g h_f = \frac{1}{3} m_A g h_i$ use $m_A = \frac{1}{2} m_B$

$\Rightarrow \frac{3}{2} m_B g h_f = \frac{1}{3}\frac{1}{2} m_B h_i$

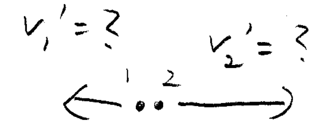
$\Rightarrow$ $h_f = \frac{1}{9} h_i$

(6) ONLY HORIZONTAL TRACK → no x,y ONLY x!

BEFORE



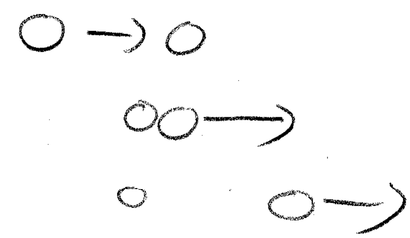
AFTER



equal masses: $m_1 = m_2 = m = \boxed{0.1}$ kg

See by intuition that one stops and the other moves on (equal masses, no friction and elastic collision (no conversion of kinetic energy into (nonconservative) heat))

Think of the pool balls:



Or do the math:

Use conservation of energy and momentum:

BEFORE

energy $K_{tot} = \frac{1}{2} m_1 v_1^2 + 0 = \frac{1}{2} m v_1^2$

momentum $p_{tot} = m_1 v_1 = m v_1$

AFTER

energy $$K'_{tot} = \frac{1}{2} m_1 (V_1')^2 + \frac{1}{2} m_2 (V_2')^2$$

$$= \frac{1}{2} m (V_1')^2 + \frac{1}{2} m (V_2')^2$$

$$= \frac{1}{2} m \left((V_1')^2 + (V_2')^2 \right)$$

momentum $$P'_{tot} = m_1 V_1' + m_2 V_2'$$

$$= m (V_1' + V_2')$$

Before = After

energy $$K_{tot} = K'_{tot}$$

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m \left((V_1')^2 + (V_2')^2 \right) \qquad \text{eq. 1)}$$

momentum $$P_{tot} = P_{tot}'$$

$$m v_1 = m (V_1' + V_2') \qquad \text{eq. 2)}$$

m cancels out in both eqns.:

$$v_1^2 = (v_1')^2 + (v_2')^2 \quad \text{from Energy eq. 1)}$$

$$v_1 = v_1' + v_2' \quad \text{from momentum eq. 2)}$$

$$\Rightarrow \quad v_1^2 = (v_1 - v_2')^2 + (v_2')^2$$

$$v_1^2 = v_1^2 - 2v_2'v_1 + 2(v_2')^2$$

$$\Rightarrow \quad 2v_2'v_1 = 2(v_2')^2$$

$$\Rightarrow \quad v_2' = v_1$$

and back into Energy eqn.:

$$v_1^2 = (v_1')^2 + (v_1)^2$$

$$\Rightarrow \quad v_1' = 0$$

glider 1 stops, 2 moves with it's speed in same direction