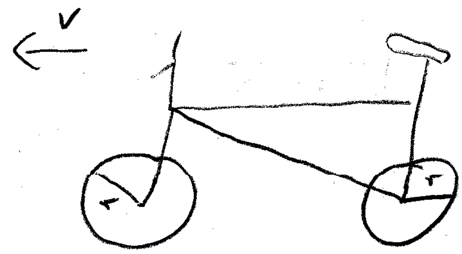
HW #10 Solutions

1



$r = \boxed{0.31\,m}$

$M_{total} = \boxed{74\,kg}$

$I_{wheel} = \boxed{0.092\,kg\,m^2}$

Coasting at v = const, what is fraction of rotational energy?

Rotational energy: $E_{ROT} = \frac{1}{2} I \omega^2$

where I: moment of inertia

ω: angular velocity

Rotational energy of the wheels (we have 2 wheels)

$$E_{ROT}^{WHEELS} = \underbrace{\frac{1}{2} I_{wheel} \omega^2}_{\text{WHEEL 1}} + \underbrace{\frac{1}{2} I_{wheel} \omega^2}_{\text{WHEEL 2}} = I_{WHEEL}\, \omega^2$$

But $\omega = \frac{v}{r}$, so

$$E_{ROT}^{WHEELS} = I_{WHEEL} \frac{v^2}{r^2}$$

Total energy:

$$E_{tot}^{Bike} = \underbrace{\frac{1}{2} M v^2}_{\text{translation of whole bike}} + \underbrace{E_{ROT}^{WHEELS}}_{\text{rotation of wheels}}$$

$$= \frac{1}{2} M v^2 + I_{WHEEL} \frac{v^2}{r^2}$$

Ratio of energies R:

$$R = \frac{E_{ROT}^{WHEELS}}{E_{total}^{bike}} = \frac{I_{WHEEL}\frac{v^2}{r^2}}{\frac{1}{2}Mv^2 + I_{WHEEL}\frac{v^2}{r^2}}$$

$$= \frac{I_{WHEEL}\frac{1}{r^2}}{\frac{1}{2}M + I_{WHEEL}\frac{1}{r^2}} \left(\frac{\cancel{v^2}}{\cancel{v^2}}\right)^{1}$$

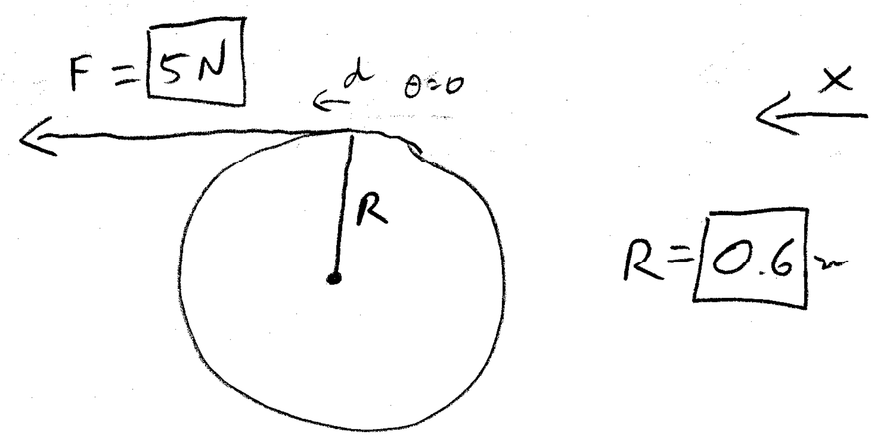
answer does not depend on v as expected!

Put in numbers:

$$R = \frac{\boxed{0.092}\,kg\,m^2\,\frac{1}{r^2}}{\frac{1}{2}\boxed{74}\,kg + \boxed{0.092}\,kg\,m^2\,\frac{1}{r^2}}$$

$$R = \boxed{0.0252}$$

2



a) How much rope unwinds while wheel makes one revolution?

- The rope is wound along outer rim of wheel of $R = \boxed{0.6\,m}$, we need to unwind

$2\pi R = 2\pi \cdot \boxed{0.6\,m} = \boxed{3.77\,m}$ of

rope for one rev. (circumference of outer rim is $2\pi R$)

b) Work done by rope on wheel?

We pulled with 5N over 3.77 m, the displacement was parallel to the pull force direction $\Rightarrow$ $W = F \cdot d \cdot \underbrace{(\cos\theta)}_{\theta=0}$

$= \boxed{5\,N} \cdot \boxed{3.77}\,m$

$W = \boxed{18.85}\,Nm$

c) Torque due to F on wheel:

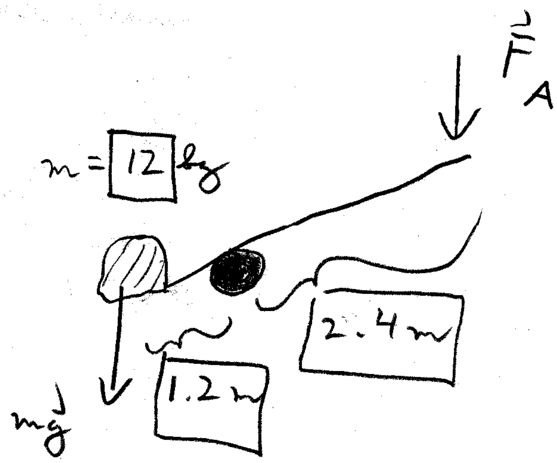
$$|\tau| = |F| \cdot |r| \cdot \sin\theta$$

$\theta = 90° = \frac{\pi}{2}$

$$= \boxed{5}N \cdot \boxed{0.6}m$$

$$|\tau| = \boxed{3}\,Nm$$

(3)



magnitude of force needed to lift rock.

Gravity acts on rock, to lift it, we have to overcome gravity. (As long as it is not lifted in the air, gravity is cancelled by normal force from ground)

$\Rightarrow$ We have to apply a big enough torque to overcome the torque by gravity

use $\tau = |F| \cdot |r| \cdot \sin\theta$

$$\tau_A = \boxed{2.4\,m}\,F_A \geq \underset{\text{rock due to gravity}}{\tau} = \boxed{1.2\,m}\cdot\underbrace{\boxed{12\,kg}\cdot 9.8\,\frac{m}{s^2}}_{mg}$$

to lift it: τ_A at least equal τ_{Rock}

$\Rightarrow$

$$\boxed{2.4\,m}\,F_A = \boxed{1.2\,m}\cdot\boxed{12\,kg}\cdot 9.8\,\frac{m}{s^2}$$

Solve for F_A:

$$\Rightarrow \quad F_A = \boxed{\frac{1}{2}}\cdot\boxed{12\,kg}\cdot 9.8\,\frac{m}{s^2}$$

$$F_A = \boxed{58.8\,N}$$

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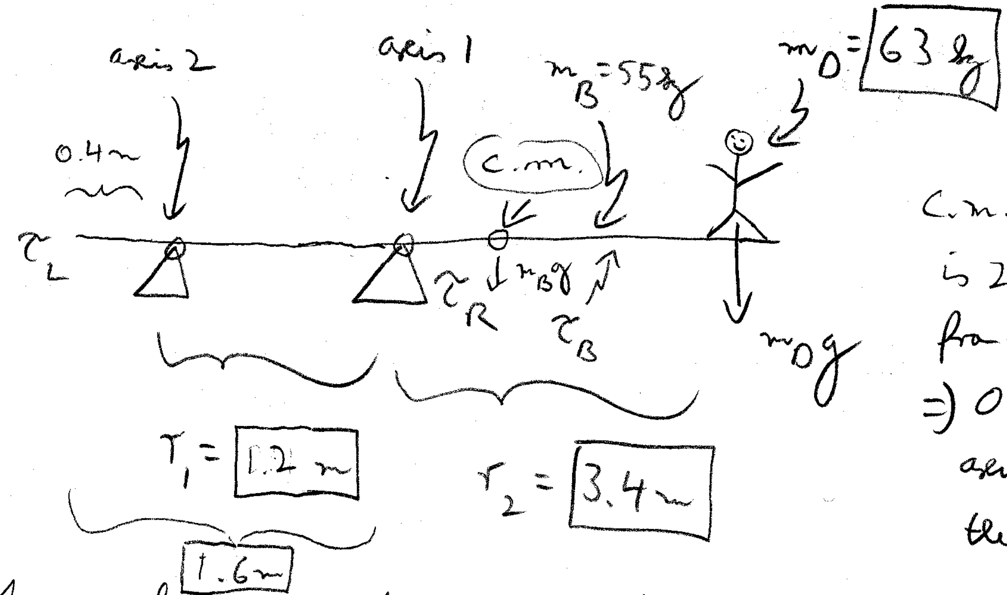


c.m. is 2.5 m from the end ⇒ 0.9 m from axis 1 to the right

Assume forces are vertical, find forces acting on board due to the two supports ⇒ equilibrium!

$\boxed{\text{AXIS 1}}$ $\boxed{\sum \tau = 0}$

— Diver exerts a torque of magnitude

$$|\tau_D| = r_2 (m_D g) = \boxed{2.1} kN$$

and negative sign on the board (cw):

$$\boxed{\tau_D = \boxed{-2.1} kN}$$

— Gravity exerts a torque of (on c.m.)

$$\tau_B = -\boxed{0.9\,m}\, m_B g$$

distance axis 1 ↔ center of mass

$$\boxed{\tau_B = -0.485\,kN}$$

on the board. This is the net torque due to the mass of the board of 55 kg being distributed uniformly along the board, and gravity pulling on the board's mass.

— Left support exerts a torque of

unknown!

$$\boxed{\tau_L = r_1 \cdot F_L = \boxed{1.2\,m} \cdot F_L}$$

on the board

Use $\sum \tau = 0$ and get F_L:

$$-2.1\,kN - 0.485\,kN + \boxed{1.2\,m} \cdot F_L = 0$$

$$\Rightarrow \quad F_L = \frac{\boxed{-2.585}\,kN}{\boxed{-1.2\,m}} = \boxed{2.15}\,kN$$

$F_L > 0$, so τ_L is positive $\Rightarrow$ CCW $\Rightarrow$ downward

$\boxed{\text{AXIS 2}}$ Do same thing for axis 2:

— $|\tau_D| = (r_1 + r_2)\, m_D\, g = \boxed{4.6\,m}\; 63\,kg\; 9.8\,m/s^2$

and CW, so

$$\boxed{\tau_0 = \boxed{-2.84}\,\text{kN}}$$

— board has center of mass $\boxed{2.1\text{ m}}$ away to the right from axis 2, so (CW)

$$\tau_B = -\boxed{2.1\text{ m}}\cdot m_B g$$

$$\boxed{\tau_B = \boxed{-1.139}\,\text{kN}}$$

— right support acts with torque

$$\tau_R = \boxed{1.2\text{ m}}\,F_R$$ unknown

on board

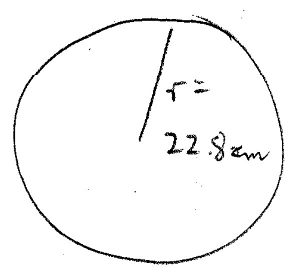
use $\sum \tau = 0$ to get

$$(-2.84 - 1.139)\,\text{kN} + \boxed{1.2\text{ m}}\,F_R = 0$$

$\Rightarrow$ $$F_R = \boxed{3.31}\,\text{kN}$$ positive $\Rightarrow$ CW $\Rightarrow$ upward

5 a)

grinding wheel



$m = \boxed{22.0\,kg}$

$\Rightarrow$ uniform cylindrical disc $\Rightarrow$ lookup I_{disc} in book table 8.1

$$I_{disc} = \frac{1}{2} M R^2 = \boxed{22.0\,Kg} \cdot \boxed{0.228\,m}^2 \cdot \frac{1}{2}$$

$$I_{disc} = 0.572\ kg\ m^2$$

b) From slowing-down process get the friction:

$$\tau_{Fr} = I_{disc}\,\alpha$$

use $\alpha t = \omega_{final} - \omega_{initial}$ to get α

$\omega_{final} = 0$, $\omega_{initial} = \dfrac{1200 \cdot 2\pi}{60\,secs}$ (1200 rpm)

$\Rightarrow \quad \alpha t = -\,20 \cdot 2\pi/s \qquad t = \boxed{59.8\,s}$

$\Rightarrow \quad \alpha = \dfrac{-40\pi}{59.8\,s^2} = \boxed{-2.1}\ \dfrac{1}{s^2}$

$$\Rightarrow \tau_{F_r} = \boxed{0.572}\,\text{kg m}^2 \left(\boxed{-2.1}\,\frac{1}{s^2}\right).$$

$$\Rightarrow \tau_{F_r} = -\boxed{1.2}\,\frac{\text{kg m}^2}{s^2} \Rightarrow |\tau_f| = \boxed{1.2}\,\frac{\text{kg m}^2}{s^2}$$

To accelerate from rest to $\boxed{1200}$ rpm, in $\boxed{3.95\,s}$, we need torque

$$\tau_{\text{speed-up}} - |\tau_f| = I_{\text{disc}}\,\alpha$$

and α must be, from

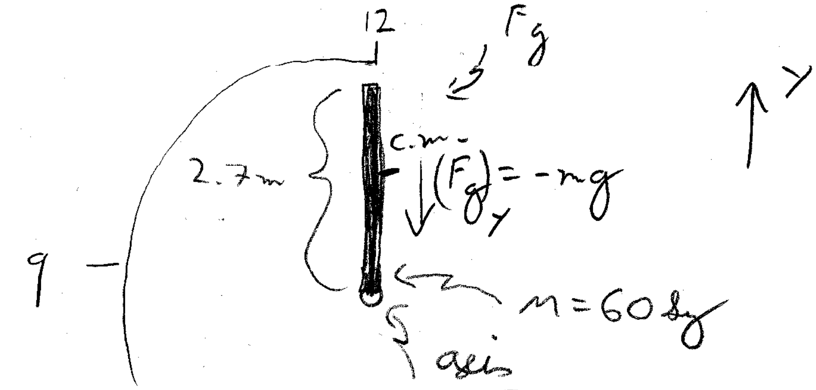
$$\alpha t = \omega_{\text{final}} - \omega_{\text{initial}}$$

$$\alpha = \frac{\omega_{\text{final}} - 0}{\boxed{3.95\,s}} = \frac{\overbrace{40\pi}^{=\omega_{\text{final}}}}{\boxed{3.95}\,s^2} = \boxed{31.8}\,\frac{1}{s^2}$$

$$\Rightarrow \tau_{\text{speed-up}} = \boxed{0.572}\,\text{kg m}^2\left(\boxed{31.8}/s^2\right) + \boxed{1.2}\,\frac{\text{kg m}^2}{s^2}$$

$$\Rightarrow \tau_{\text{speed-up}} = 19.4\,\frac{\text{kg m}^2}{s^2} \quad [\text{N m}]$$

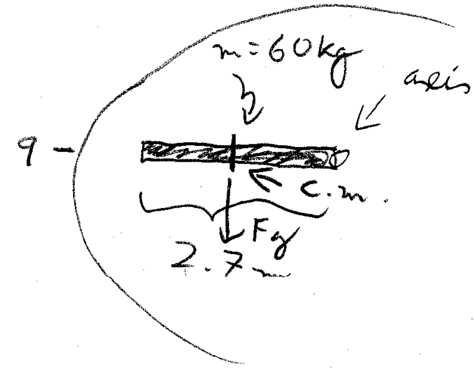
(6) a) at NOON



at noon, gravity force on hour hand and line from axis of rotation to c. mass where gravity acts on hand are parallel $\Rightarrow \Theta = 0 \Rightarrow \sin\Theta = 0$

$\Rightarrow$ no torque, as $|\vec{\tau}| = |F||r| \sin\Theta$!

b) at 9.00 am



The torque due to the weight now is, assuming a uniform mass distribution along the hand the c.m. is $\boxed{1.35\ m}$ to the left of the axis

$\Rightarrow |\tau_{at\ 9.00}| = \boxed{1.35\ m} \cdot 60\ kg \cdot 9.8\ \frac{m}{s^2}$

$$\tau_{at\,9\,am} = \boxed{1.35}\,m\ \boxed{60\,kg}\ 9.8\,\frac{m}{s^2}$$

$$\tau = 794\ Nm$$

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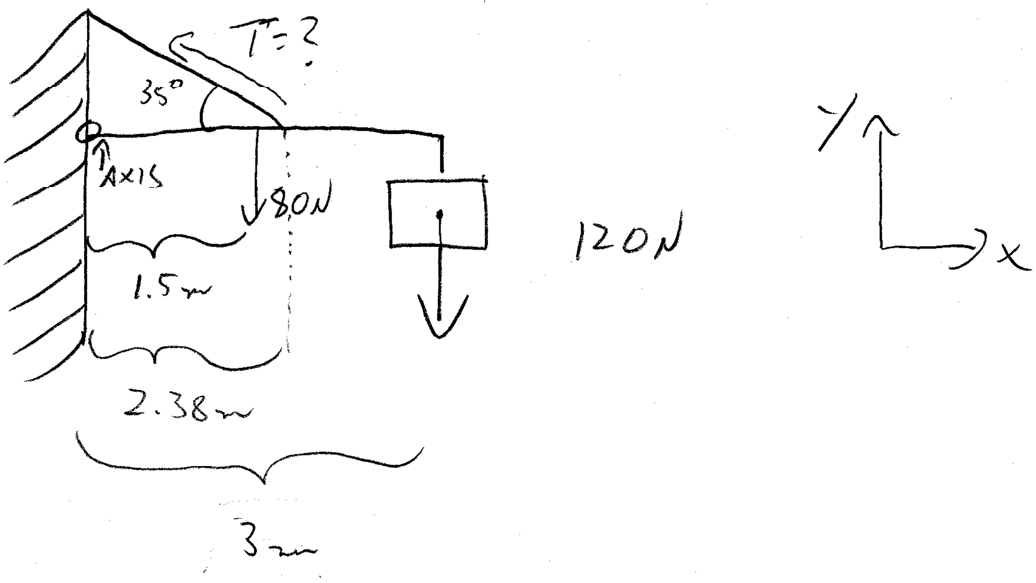


a) T=? Choose rotation axis on the hinge

The vertical component of T must cancel the other torques:

$$\tau_T = 2.38m \cdot T \sin 35^\circ$$

$$\tau_{cm} = -1.5m \; 80N$$

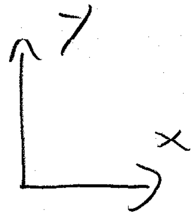
$$\tau_{crate} = -3m \; 120N$$

equilibrium: $\sum \tau = 0$

$$\Rightarrow \quad 2.38m \cdot T \sin 35^\circ = 1.5m \; 80N + 3m \; 120N$$

$$\Rightarrow \quad T = \frac{1.5m \; 80N + 3m \; 120N}{2.38m \sin 35^\circ} = 351\,N$$

6) $F_x = ?$, $F_y = ?$



equilibrium $\Rightarrow \sum \vec{F} = 0$

$\boxed{\text{in } x}$ $\quad T_x = -F_x \qquad \boxed{T_x = -T\cos\theta}$

$\Rightarrow -351N \cos 35 = -F_x$

$\Rightarrow$ $F_x = 288N$

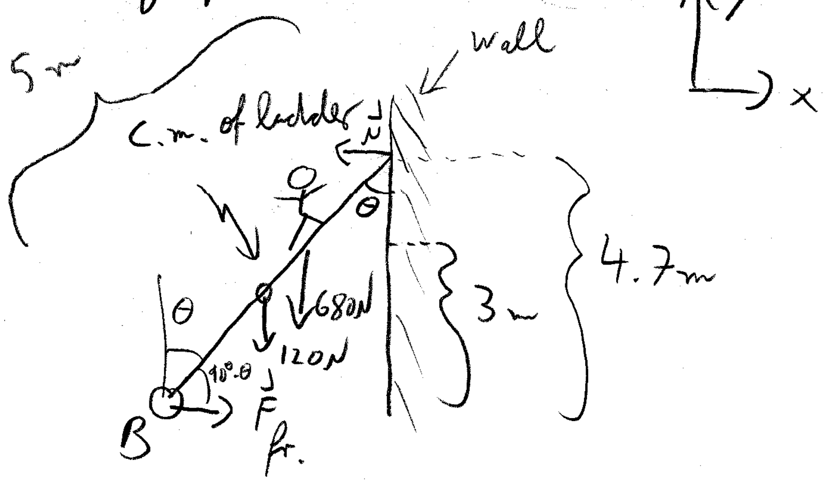
$\boxed{\text{in } y}$

$-80N - 120N + T_y = -F_y$

$\Rightarrow -200N + 351N \sin 35^\circ = -F_y$

$\Rightarrow$ $F_y = -1.32 \approx -2$

(8) Just like the group problem



choose B as axis of rotation, equilibrium $\Rightarrow$

$$\sum \tau = 0$$

$$\sum F = 0$$

first, find θ: $\cos\theta = \frac{4.7\text{ m}}{5\text{ m}}$

$$\theta = \arccos\left(\frac{4.7}{5}\right)$$

$$\theta = 19.9°$$

then, find τ's:

distance axis $\to$ person

$$|\tau_{Person}| = |\tau_p| = 680N \cdot d_{BP} \cdot \sin\theta$$

$$\text{But } d_{BP} = \frac{3\text{ m}}{\sin(90°-\theta)} = 3.1915\text{ m}$$

CW!

So $|\tau_p| = 738\text{ Nm} \Rightarrow \boxed{\tau_p = -738\text{ Nm}}$

$$\tau_{ladder} = -120\,N \cdot 2.5\,m \sin\theta$$

$$\boxed{\tau_{ladder} = -102\,Nm}$$ CW!

$$\tau_{wall} = |N| \cdot 5\,m \cdot \sin(90° - \theta)$$

$$\boxed{\tau_{wall} = |N| \cdot 4.7\,m}$$ and $\tau_{friction} = 0$ as $r = 0$

CCW!

$$\sum \tau = 0 \Rightarrow$$

$$|N| \cdot 4.7\,m - 102\,Nm - 738\,Nm = 0$$

$$\Rightarrow |N| = \frac{102\,Nm + 738\,Nm}{4.7\,m} = 178\,N$$

We know from picture that $\vec{N}$ points horizontally to the left, so

$$N_x = -178\,N$$

But from $\sum F_y = 0$

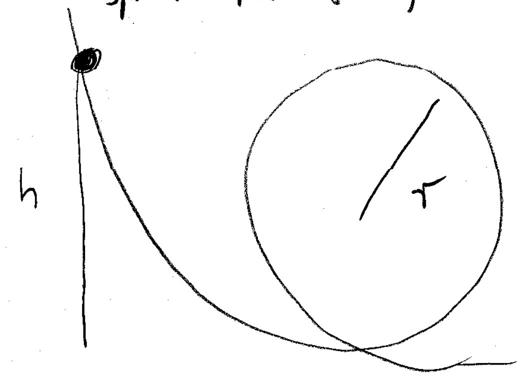
we see ~~that~~ $\vec{F}_{Fr} = -\vec{N}$, and F_{Fr}, N horizontal

$$\Rightarrow \quad F_{Fr_x} = -N_x$$

$$\Rightarrow \quad F_{Fr_x} = 178\,N$$ (, x points to the right)

$$\Rightarrow \quad F_{Fr} \approx 180\,N$$ to the right

sphere radius R, mass M



a) Find minimum value of h so that sphere always remains on track → Sliding sphere → no rotation!

At the top, the sphere's speed must at least equal the speed we get if only g is the centripetal acceleration

$$M\frac{v^2}{r} \geq Mg \Rightarrow v^2 = gr$$

So it's kinetic energy must be at least $\frac{1}{2}mv^2 = \frac{1}{2}mgr$

All the kinetic energy it has at the top of the loop must come from it's initial potential energy.

Before it starts sliding: $E_i = E_{pot_i} = Mgh$

at the top of the loop: $E_f = E_{pot_f} + E_{kin_f}$

$$E_i = E_f$$

$$\Rightarrow \quad Mgh = E_{pot_f} + E_{kin_f}$$

$$E_{pot_f} = Mg2r$$

$$E_{kin_f} = \frac{1}{2} Mgr \quad \text{(from requirement from before)}$$

$$\Rightarrow \quad Mgh = Mg2r + \frac{1}{2} Mgr$$

$$\Rightarrow \quad h = \frac{5}{2} r$$

b) Now it is rolling, so we have rotational energy also. Again, it's speed must satisfy, at top of loop:

$$m\frac{v^2}{r} = mg \Rightarrow v^2 = gr$$

$$\omega^2 = \frac{v^2}{r^2}$$

But now $E_{kin_f} = \frac{1}{2} Mv^2 + \frac{1}{2} I_{sphere} \omega^2$

$$= \frac{1}{2} Mgr + \frac{1}{2} \frac{2}{5} MR^2 \frac{V^2}{R^2}$$

$$E_{kin_f} = \frac{1}{2} \; gr + \frac{1}{2}\frac{2}{5} M R^2 \frac{gr}{R^2}$$

$$= \frac{1}{2} \; gr + \frac{1}{5} M g r$$

$$E_{pot_f} = M g 2r \quad \text{(still)}$$

$$E_i = E_{pot_i} = M g h \quad \text{(still)}$$

$$\Rightarrow \quad E_i = E_f \Rightarrow M g h = \underbrace{M g 2r}_{E_{pot}} + \underbrace{\frac{1}{2} M g r}_{E_{TRANS}} + \underbrace{\frac{1}{5} M g r}_{E_{ROT}}$$

$$\Rightarrow \quad h = 2r + \frac{1}{2} r + \frac{1}{5} r$$

$$\Rightarrow \quad h = \left(\frac{5}{2} + \frac{1}{5}\right) r = \left(\frac{25}{10} + \frac{2}{10}\right) r = \frac{27}{10} r$$

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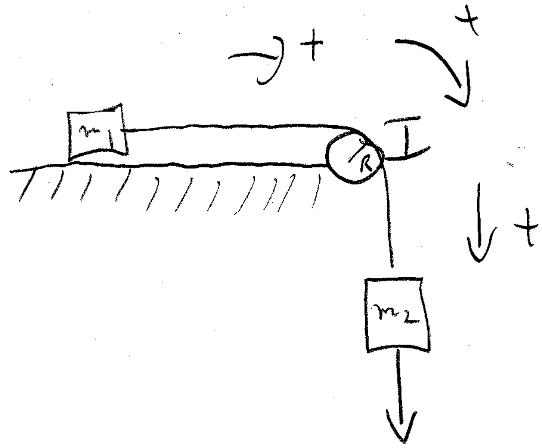
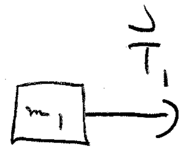


no friction! acceleration $a = ?$

-ve as α is positive for ccw, but +ve a for cw

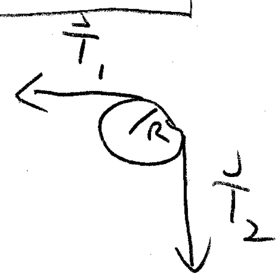
use $F = ma$ and $\tau = I\alpha = I\frac{-a}{R}$

m_1:



$$\boxed{T_1 = m_1 a}$$

Pulley

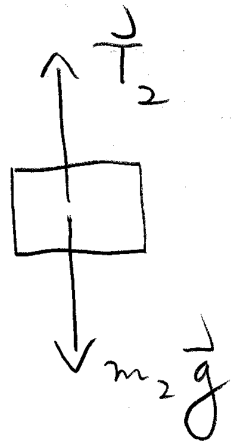


$$\sum \tau = I\alpha = -I\frac{a}{R}$$

$$RT_1 - RT_2 = -I\frac{a}{R}$$

$$\Rightarrow \boxed{T_2 = T_1 + I\frac{a}{R^2} = m_1 a + \frac{I}{R^2}a}$$

$\boxed{m_2}$



$$m_2 g - T_2 = m_2 a$$

But now we know $T_2 = m_1 a + \frac{I}{R^2} a$

$$\Rightarrow \quad m_2 g - m_1 a - \frac{I}{R^2} a = m_2 a$$

$$\Rightarrow \quad m_2 g = \left(m_2 + m_1 + \frac{I}{R^2}\right) a$$

$$\Rightarrow \quad a = \frac{m_2 g}{m_2 + m_1 + \frac{I}{R^2}}$$

(11) in pike position:

diver has $I_{PIKE} = 15.5 \text{ kg m}^2$

in tucked position:

diver has $I_{TUCKED} = 8.0 \text{ kg m}^2$

Initial ang. mom.:

$$L_i = 106 \text{ kg } \frac{m^2}{s}$$

This momentum stays the same as there are never any torques acting on the diver:

$$L_i = 106 \text{ kg } \frac{m^2}{s} = L$$

a) How many turns can be made in a tucked position?

$$L = I\omega \Rightarrow \omega = \frac{L}{I}$$

But $\omega = 2\pi \frac{\text{number of turns}}{\text{total time take}}$ if L and thus

ω is constant.

So $\omega = \frac{L}{I} = 2\pi \frac{N}{\Delta t}$

But then

$$N = \frac{L}{I} \frac{1}{2\pi} \Delta t$$

$\Rightarrow$ we need Δt:

He jumps of 10 m platform:

$a = -g$ $\uparrow y$

$$\Delta y = -10\,m = \frac{1}{2}(-9.8\,m/s^2)\Delta t^2$$

$$\Rightarrow \quad \Delta t = \sqrt{2\frac{10}{9.8}s^2} \left(= \sqrt{2g\Delta y}\right.$$

$$\Delta t = 1.43\,s$$

So then $N = \frac{L}{I}\frac{1}{2\pi}\Delta t = \frac{106\,kg\,m^2/s}{8.0\,kg\,m^2}\frac{1}{2\pi}1.43\,s = 3.01$

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b) In the $\boxed{\text{pike}}$ position we have $I = 15.5\,kg\,m^2$

$$N = \frac{L}{I_{PIKE}} \frac{1}{2\pi} \Delta t = \frac{106\,kg\,\frac{m^2}{s}}{15.5\,kg\,m^2} \frac{1}{2\pi} 1.43\,s$$

$$N = 1.6$$