

Prob 11 - Bonus Prob Solution

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- ① Maximum pressure that is survivable : 1000 times atmospheric pressure.

What is the maximum depth such organism can withstand in the sea?

$$P_{\max} = \boxed{1000} \text{ atm}$$

$$1 \text{ atm} = 101.3 \text{ kPa} \quad (\text{from book page 303})$$

$$\Rightarrow P_{\max} = \boxed{101300} \text{ kPa}$$

$$= \boxed{1.013 \cdot 10^8} \text{ Pa}$$

We know that pressure in the ocean depends on depth according to $p = \rho g h$, where h is the depth and ρ is density of water

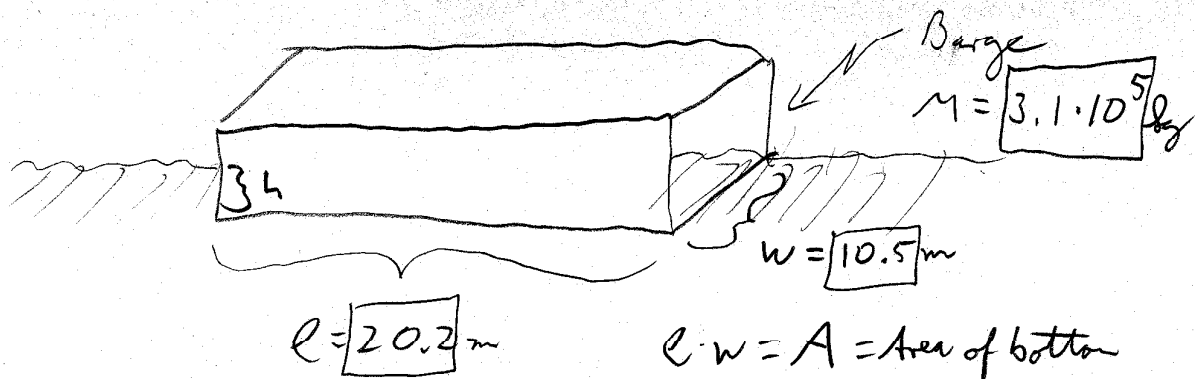
$$\Rightarrow P_{\max} = \rho g h, \quad \rho = 1027 \frac{\text{kg}}{\text{m}^3}$$

$$\Rightarrow h = \frac{P_{\max}}{\rho g}$$

$$\Rightarrow h = \frac{\boxed{1.013 \cdot 10^8} \text{ Pa}}{1027 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \frac{\text{m}}{\text{s}^2}} \quad \left(1 \text{ Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2} \right)$$

$$h = \boxed{10065} \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2}}{\frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}}{\text{s}^2}} \approx \boxed{10.1 \cdot 10^3} \text{ m} = \boxed{10.1} \text{ km}$$

(2)



The weight of the barge must equal the weight of the displaced water!

We can see this from ($P = P_{\text{water}}$), ($h = \text{depth}$), ($A = \text{Area of barge bottom}$)

$$\rho g h = P_{\text{pressure}} = \frac{F_{\text{on B by Water}}}{A} \quad \leftarrow \begin{array}{l} \text{this must cancel gravity} \\ (\text{mg}) \end{array}$$

multiply by A : ($h \cdot A = \text{Volume of displaced water}$)

$$\rho g V_{\text{displaced water}} = F_{\text{on barge by Water}} \stackrel{! \leftarrow \text{for it to swim!}}{=} Mg$$

So:

weight of displaced water = weight of barge

Volume of displaced water!

$$W_w = W_B$$

$$\rho_{\text{water}} g V = Mg$$

$$\left(\text{but } m_{\text{water}} = \rho \cdot V_{\text{water}}, \text{ so } \rho_{\text{water}} g V = m_{\text{water}} g = \underline{\text{weight}} \right)$$

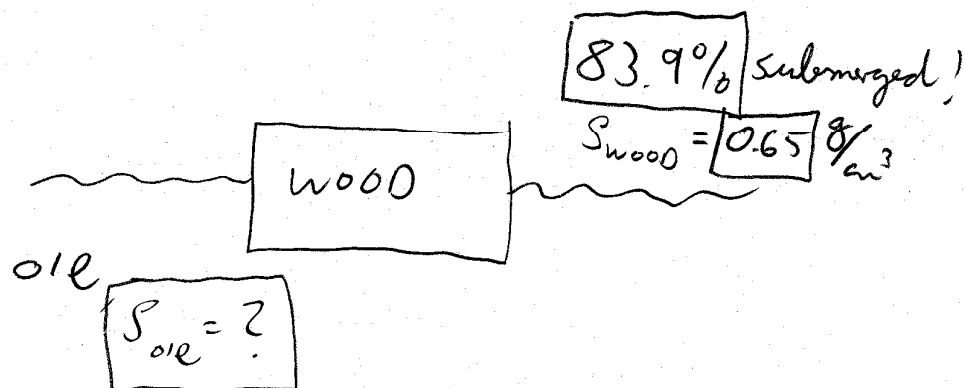
$$\Rightarrow V = \frac{M}{\rho} \quad \left(\text{but } V = l \cdot w \cdot h \right) \quad \rho_{\text{water}}$$

$$\Rightarrow h = \frac{M}{l \cdot w \cdot \rho} = \frac{M}{A \cdot \rho} = \frac{3.1 \cdot 10^5 \text{ kg}}{20.2 \text{ m} \cdot 10.5 \text{ m} \cdot 10^3 \frac{\text{kg}}{\text{m}^3}}$$

$$\Rightarrow h \approx \boxed{1.5} \text{ m}$$

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(3)



The ratio of densities is equal to the ratio of Volumes:

$$\frac{S_{\text{wood}}}{S_{\text{oil}}} = \frac{V_{\text{submerged}}}{V_{\text{total}}} = \boxed{0.839}$$

83.9% = 0.839!

$$\Rightarrow S_{\text{wood}} = 0.839 S_{\text{oil}}$$

$$\Rightarrow S_{\text{oil}} = \frac{1}{\boxed{0.839}} S_{\text{wood}} = \frac{1}{\boxed{0.839}} \boxed{0.65} \text{ g/cm}^3$$

$$\Rightarrow S_{\text{oil}} = \boxed{0.775} \text{ g/cm}^3$$

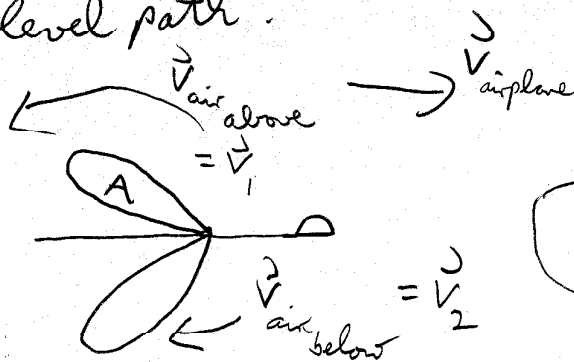
4

Airplane, level path:

$$P_{\text{above}} = P_1$$

$$P_1 < P_2$$

$$P_{\text{below}} = P_2$$



$$V_2 = 80.8 \text{ m/s}$$

$$A = 190 \text{ m}^2$$

$$P_2 - P_1 = \Delta P = 540 \text{ Pa}$$

$$V_2 < V_1$$

it flies because the pressure above the wings is less than the pressure below the wings, which results in an upward net force on the wings, that cancels the downward gravity force!

For the pressure above the wings to be less than below the air must flow by faster above the wings than below the wings!

b) $V_1 = ?$, use Bernoulli's law: (2: below, 1: above)

$$\frac{1}{2} \rho V_1^2 + \frac{1}{2} \rho V_2^2 = \Delta P$$

where $\Delta P = \text{pressure difference} = P_2 - P_1 = 540 \text{ Pa}$

and V_2 given as $V_2 = 80.8 \text{ m/s}$

and $\rho = \rho_{\text{air}} = 1.29 \text{ kg/m}^3$ (from book page 307)
(up there it is 0°C)

$$\Rightarrow V_1^2 = \left(540 \text{ Pa} + \frac{1}{2} (1.29 \text{ kg/m}^3) (80.8 \text{ m/s})^2 \right) \cdot 2 \frac{1}{1.29 \text{ kg/m}^3}$$

$$\Rightarrow V_1^2 = 7365.85 \frac{\text{m}^2}{\text{s}^2}$$

$$\Rightarrow V_1 = 85.8 \text{ m/s}$$

a) $W_{\text{plane}} = ?$

Pressure times area is force, the force due to ΔP must cancel gravity.

Force on wings due to ΔP :

$$\Delta P = 540 \text{ Pa} = \frac{F}{A} \Rightarrow F = 540 \text{ Pa} \cdot A$$

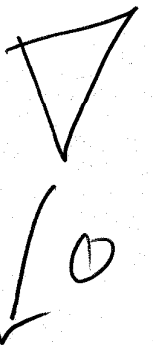
$$\Rightarrow F = 540 \text{ Pa} \cdot 2 \cdot 190 \text{ m}^2$$

$$\Rightarrow F = 2.052 \cdot 10^5 \text{ N}$$

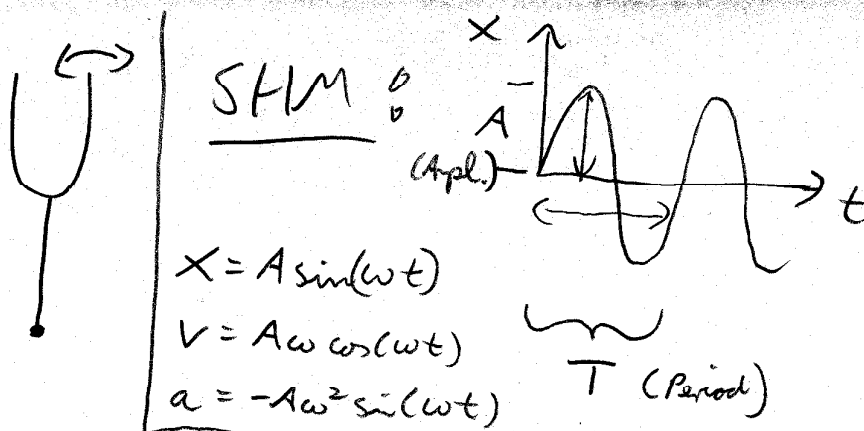
This is the force due to ΔP and this must be equal to the force due to gravity, which is the weight of the plane.

$$\Rightarrow W_{\text{plane}} = 2.052 \cdot 10^5 \text{ N}$$

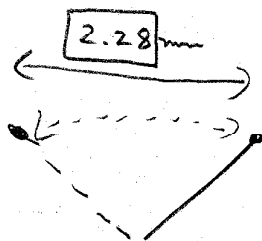
Mistake on Webermagn:
They used only one wing
and thus the answer they
want is $1.03 \cdot 10^5 \text{ N}$ (4)



(5)



Prong moves 2.28 mm between far left and far right positions:



$\Rightarrow$ Amplitude of SHM is one half of that (Amplitude is measured from middle to extreme, not from extreme to extreme) $A = 1.14$ mm

frequency of prong $f = 440.2$ Hz

$\rightarrow$ find $v_{\max}$ and $a_{\max}$

$$\omega = 2\pi f$$

$\Rightarrow$ from book $v_{\max} = \text{Amplitude} \cdot \omega$

$$= 1.14 \text{ mm} \cdot 2\pi \cdot 440.2 \text{ Hz}$$

$$v_{\max} = 3153 \frac{\text{mm}}{\text{s}} = 3.15 \frac{\text{m}}{\text{s}}$$

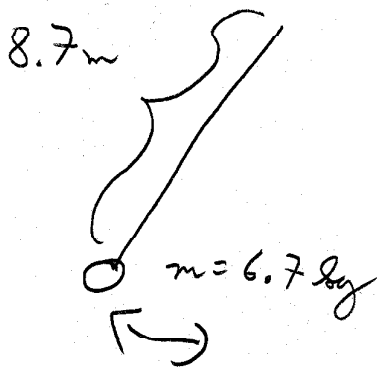
from book $a_{\text{max}} = \text{Amplitude} \cdot \omega^2$

$$= \boxed{1.14} \text{ mm} \cdot \left(2\pi \boxed{440.2} \text{ Hz} \right)^2$$

$$= 8720967.43 \text{ mm/s}^2$$

$$a_{\text{max}} \approx 8720 \text{ m/s}^2 //$$

⑥



note: indep. of mass!

$T = ?$, pendulum, from lecture

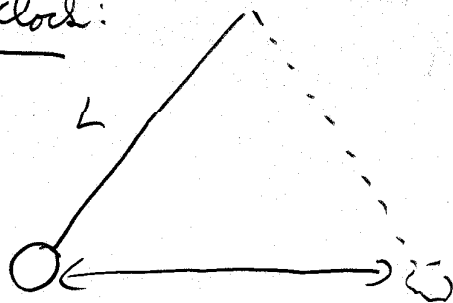
$$\omega_{\text{pend.}} = \sqrt{\frac{g}{L}}$$

but $\omega = 2\pi f = \frac{2\pi}{T}$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{\boxed{8.7} \text{ m}}{9.8 \text{ m/s}^2}} = \boxed{5.92} \text{ s} //$$

7) Grandfather clock:



amplitude



from one side to other: $\boxed{24} \text{ mm} \Rightarrow A = \boxed{12} \text{ mm}$

a) assume SHM, use relation between A and v_{max} : $\boxed{1} \text{ s} \Rightarrow T = \boxed{2} \text{ s}$

$$v_{\text{max}} = A\omega = A2\pi f = A \cdot 2\pi \frac{1}{T}$$

$$\Rightarrow v_{\text{max}} = \boxed{12} \text{ mm} \cdot 2\pi \cdot \frac{1}{\boxed{2} \text{ s}} = \boxed{37.7} \frac{\text{mm}}{\text{s}} = \boxed{3.8} \frac{\text{cm}}{\text{s}}$$

The period is two sec. because in one period, it has to go from left to right AND back!

▽ mistake in my Webernig solution

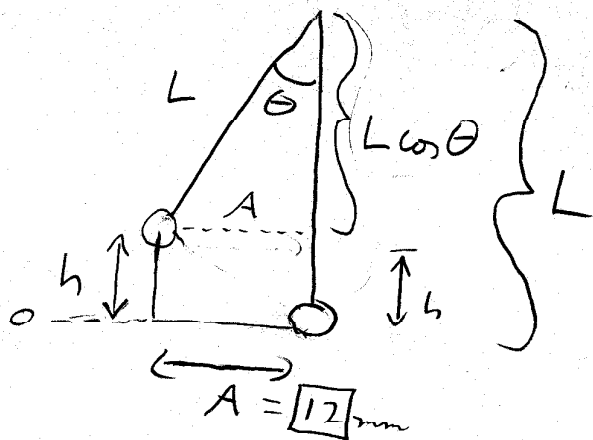
○ they want $\boxed{1.51} \text{ cm/s}$ as answer, which is obviously wrong!

b) Use energy conservation:

At it's point furthest to the left, it has zero speed and only potential energy ($= mgh$); at it's bottom point, it

has MAX speed and only kinetic energy ($= \frac{1}{2}mv^2$)

(Here, I choose my zero-point of potential E. at the bottom)



From the picture, we see that $L - h = L \cos \theta$

so $h = L(1 - \cos \theta)$

- Get L from $T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = \frac{T^2}{(2\pi)^2} g = 0.992948 \text{ m}$

- Get θ from $A = L \sin \theta$, we can only solve this pendulum for small θ ; this becomes ($\sin \theta \approx \theta$)

$$A \approx L \theta \Rightarrow \theta \approx \frac{A}{L} = \frac{12 \cdot 10^{-3} \text{ m}}{0.992948 \text{ m}} = 1.21 \cdot 10^{-2} \text{ rad}$$

then $h = L(1 - \cos \theta) = 0.992948 \text{ m} (1 - \cos(1.21 \cdot 10^{-2} \text{ rad}))$

$\Rightarrow h = 7.27 \cdot 10^{-5} \text{ m}$

now use E. conservation

same as a) ↓

$$mgh = \frac{1}{2} m v_{\text{max}}^2 \Rightarrow v_{\text{max}} = \sqrt{2gh} = 0.0377 \frac{\text{m}}{\text{s}} = 3.8 \frac{\text{cm}}{\text{s}}$$