Math 251 Worksheet 1-Section 13.2 Name________________________

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There are 3 line integral formulas.

I. $\int_C f(x, y, z)ds$ (Evaluation of a line integral as a definite integral.)

II. $\int_C (f(x, y, z)dx + g(x, y, z)dy + h(x, y, z)dz)$ (Evaluating a line integral in differential form.)

III. $\int_C \mathbf{F} \cdot d\mathbf{r}$ (Evaluating a vector-valued function over a curve C.)

Each involve integrating over a curve (NOT a region). Sometimes you will have to determine the parameterization of C. Sometimes not.

Case I. Here you need C written as a parameterized curve. Put f in terms of t and $ds = \sqrt{(x')^2 + (y')^2 + (z')^2}dt$.

Example. $\int_c x ds$ $C : x = 1 - t, y = (1 - t)^2$ $0 \le t \le 1$

Step 1. Write the integrand in terms of t: $x = (1 - t)$

Step 2. Compute $\sqrt{(x')^2 + (y')^2} = \sqrt{(-1)^2 + (2(1 - t))^2}$

Step 3. Evaluate $\int_0^1 x(t)\sqrt{(x')^2 + (y')^2}dt = \int_0^1 (1 - t)\sqrt{1 + 4(1 - t)^2}dt$

A. $\int_C (x + 2)ds$ $C : x = t$ $y = \frac{4}{3}t^{3/2}$ $z = \frac{t^2}{2}$ $0 \le t \le 2$

Step 1. Write the integral in terms of t.

Step 2. Compute $\sqrt{(x')^2 + (y')^2 + (z')^2}$

Step 3. Evaluate integral.

B. $\int_C (x^2 + y^2)ds$ C : line from (0,0) to (3,0).

Step 1. Draw C.

Step 2. Parameterize C.

Step 3. Write integrand in terms of t.

Step 4. Compute $\sqrt{(x')^2 + (y')^2}$

Step 5. Evaluate integral.

Case II. Now you want to evaluate a line integral in differential form.

Rewrite $\int_C \left[f(x(t), y(t), z(t)) \frac{dx}{dt} + g(x(t), y(t), z(t)) \frac{dy}{dt} + h(x(t), y(t), z(t)) \frac{dz}{dt} \right] dt$

Example. $\int_C (xydx + y\ dy) \quad C: x = 4t \quad y = t \quad 0 \leq t \leq 1$

Step 1. Compute $\frac{dx}{dt}, \frac{dy}{dt}$ here $\frac{dx}{dt} = 4, \frac{dy}{dt} = 1$

Step 2. Write integrand in terms of $t: (xy\frac{dx}{dt} + y\ \frac{dy}{dt}) = (4t)(t)(4) + t(1)$

Step 3. Evaluate: $\int_0^1 (16t^2 + t)dt$

A. $\int_C xydx + ydy \quad C: x = 4\cos t \quad y = 4\sin t \quad 0 \leq t \leq \frac{\pi}{2}$

Step 1. Compute $\frac{dx}{dt}, \frac{dy}{dt}$.

Step 2. Write integrand in terms of t.

Step 3. Evaluate integral.

B. $\int_C (x^2 y dx + (y - z) dy + xyz dz) \quad C : x = t \quad y = t^2 \quad z = 2 \quad 0 \leq t \leq 2$

Step 1. Compute $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$.

Step 2. Write integrand in terms of t.

Step 3. Evaluate integral.

C. $\int_C (-y dx - x dy) \quad C : x = 2\cos t \quad y = 2\sin t \quad 0 \leq t \leq \frac{\pi}{2}$

Step 1. Compute $\frac{dx}{dt}, \frac{dy}{dt}$.

Step 2. Write integrand in terms of t.

Step 3. Evaluate integral.

Case III. Here **F** and $d\mathbf{r}$ are vectors.
Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ if

A. $\mathbf{F} = 3x\mathbf{i} + 4y\mathbf{j}$ $\quad \mathbf{r} = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$ $\quad 0 \leq t \leq \frac{\pi}{2}$

Step 1. $x = 2\cos t \quad y = 2\sin t$ (x and y are the components of **r**). Write **F** in terms of t.

Step 2. Compute $\mathbf{r}'$.

Step 3. Compute $\mathbf{F} \cdot \mathbf{r}'$.

Step 4. Evaluate integral $\int_0^{\pi/2} \mathbf{F} \cdot \mathbf{r}' dt$.

B. $\mathbf{F} = 3y\,\mathbf{i} + 4x\,\mathbf{j}$ $\quad \mathbf{r} = \sqrt{4 - t^2}\,\mathbf{i} + t\,\mathbf{j}$ $\quad -2 \leq t \leq 2$

Step 1. $x = t \quad y = \sqrt{4 - t^2}$. Write **F** in terms of t.

Step 2. Compute $\mathbf{r}'$.

Step 3. Compute $\mathbf{F} \cdot \mathbf{r}'$.

Step 4. Evaluate integral $\int_2^{-2} \mathbf{F} \cdot \mathbf{r}' dt$.

C. $\mathbf{F}= x^2\ \mathbf{i}+y^2\ \mathbf{j}\ +z^2\ \mathbf{k}\quad \mathbf{r}= (\sin t)\ \mathbf{i}\ +(\cos t)\ \mathbf{j}\ +t^2\ \mathbf{k}\quad 0 \le t \le \frac{\pi}{2}$

Step 1. $x = \sin t \quad y = \cos t \quad z = t^2$. Write $\mathbf{F}$ in terms of t.

Step 2. Compute $\mathbf{r}'$.

Step 3. Compute $\mathbf{F}\cdot\mathbf{r}'$.

Step 4. Evaluate integral $\int_0^{\pi/2} \mathbf{F}\cdot\ \mathbf{r}' dt$.

The following vector-valued functions are conservative. Find f such that $\mathbf{F}= \nabla f$.

$\mathbf{F} = \frac{1}{2}xy\ \mathbf{i}\ +\frac{1}{4}x^2\ \mathbf{j}$

$\mathbf{F} = 2xy\ \mathbf{i}\ +(x^2 - y)\ \mathbf{j}$

$\mathbf{F} = e^x(\sin y\ \mathbf{i}\ + \cos y\ \mathbf{j})$

$\mathbf{F} = 2xy\ \mathbf{i}\ +(x^2 + z^2)\ \mathbf{j}\ +2zy\ \mathbf{k}$

$\mathbf{F} = e^x \cos y\ \mathbf{i}\ -e^x \sin y\ \mathbf{j}\ +2\ \mathbf{k}$