Math 251 Worksheet for 13.4 Name________________________

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16.4 deals with line integrals in differential form and the curve C is closed. The idea is that parameterizing may be difficult and sometimes the integration is difficult as well (see second example.)

Evaluate $\int_C Pdx + Qdy$ for the given curves.

Example 1. $\int_C (y-x)dx + (2x-y)dy$

C: boundary of the region lying between the graphs of $y = x$ and $y = x^2 - x$.

Step 1. Draw region. Here: *See Graph file.

Step 2. Is it closed? Here, yes.

Step 3. Identify P and Q; here $P = y - x,\ Q = 2x - y$.

Step 4. Compute $\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 2 - 1 = 1$

Step 5. Evaluate. Here $\int_R \int \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \right) dA.$ $\int_0^2 \int_{x^2-x}^{x} 1dy\, dx = \int_0^2 (x - (x^2 - x))dx =$ $\int_0^2 (2x - x^2 dx = x^2 - \dfrac{x^3}{3} = 4 - \dfrac{8}{3} = \dfrac{4}{3}.$

Example 2: Compute $\int_C (\arctan x + y^2)dx + (e^y - x^2)dy$

C: bounded by the curves $y = 0\ (-3 \leq x \leq -1,\ 1 \leq x \leq 3)$ above by $x^2 + y^2 = 1$ below by $x^2 + y^2 = 9$.

Step 1. Draw region. Here *See Graph file.

Step 2. Is it closed? Here, yes.

Step 3. Identify P and Q. Here $P = \arctan x + y^2 \qquad Q = e^y - x^2$.

Step 4. Compute $\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = -2x - (2y)$.

Step 5. Evaluate. Here $\int_R \int \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} \right) dA. = \int_R \int -2(x+y)dA$

It is easier to do this problem in polar coordinates. $\int_0^{\pi}\int_1^3 -2(\mathbf{r}\cos\theta+\mathbf{r}\sin\theta)\mathbf{r}d\mathbf{r}d\theta =$
$-2\int_0^{\pi}\int_1^3 \mathbf{r}^2(\cos\theta+\sin\theta)d\mathbf{r}d\theta = -2\left[9-\frac{1}{3}\right](2)$

A. $\int_C xe^y dx + e^x dy$ C: boundary of the region lying between the graph of $y = x$ and $y = x^2$.

Step 1. Draw region. *See Graph file.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q.

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$.

B. $\int_C 2xydx + (x+y)dy$ C: boundary of the region lying between the graphs of $y = 0$ and $y = 4 - x^2$.

Step 1. Draw region.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q.

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$.

C. $\int_C 2\tan\frac{y}{x}dx + \ln(x^2+y^2)dy \quad C: x = 4 + 2\cos\theta \qquad y = 4 + \sin\theta$

Step 1. Draw region.

Step 2. Is it closed? If yes, proceed.

Step 3. Identify P and Q.

Step 4. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Step 5. Evaluate $\int_R\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$.