Math 251 Worksheet for 13.9 Name________________________

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Divergence Theorem: Let E be a simple solid region and the S be the boundary of the surface of E, given with positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. Then

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\cdot dV$$

Divergence Theorem involves
1) Closed surface.
2) Vector-valued integrand.

Example 1. Evaluate: $\iint_S \mathbf{F}\cdot d\mathbf{S}$ when $\mathbf{F} = x\mathbf{i}+y^2\mathbf{j}+z\mathbf{k}$. E is the solid region bounded by the coordinate planes and the plane $2x+2y+z=6$.

Step 1. Is the region closed? Here, yes.

Step 2. Is the integrand a vector-valued function? Here, yes.

Step 3. Identify $\mathbf{F}$. Here $\mathbf{F} = x\mathbf{i}+y^2\mathbf{j}+z\mathbf{k}$.

Step 4. Compute div $\mathbf{F}$. Here div$\mathbf{F} = 1+2y+1$.

Step 5. Evaluate $\iiint_E$ div $\mathbf{F}dV$. Here $\iiint_E \text{div } \mathbf{F}dV = \int_0^3\int_0^{3-y}\int_0^{6-2x-2y}(2+2y)dzdxdy$

Example 2: Evaluate: $\iint_S \mathbf{F}\cdot d\mathbf{S}$ when $\mathbf{F} = x^2\mathbf{i}-2xy\mathbf{j}+xyz^2\mathbf{k}$. E is the solid region bounded $z=\sqrt{9-x^2-y^2}$, $z=0$.

Step 1. Is the region closed? Here, yes.

Step 2. Is the integrand a vector-valued function? Here, yes.

Step 3. Identify $\mathbf{F}$. Here $\mathbf{F} = x^2\mathbf{i}-2xy\mathbf{j}+xyz^2\mathbf{k}$.

Step 4. Compute div $\mathbf{F}$. Here div$\mathbf{F} = 2x-2x+2xyz$.

Step 5. Evaluate $\iiint_E$ div $\mathbf{F}dV$. Here $\iiint_E \text{div } \mathbf{F}dV = \int_0^{2\pi}\int_0^3\int_0^{\sqrt{9-r^2}} 2xyz\,dz\,rdr\,d\theta$

Problem 1. Evaluate $\mathbf{F} = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$. E is the solid region bounded by $z = \sqrt{4 - x^2 - y^2}$ $z = 0$.

Step 1. Is the region closed?

Step 2. Is the integrand a vector-valued function?

Step 3. Identify $\mathbf{F}$.

Step 4. Compute div $\mathbf{F}$.

Step 5. Evaluate $\iiint_E \text{div } \mathbf{F} dV$.

Problem 2. Evaluate $\mathbf{F} = xyz\mathbf{j}$. E is the solid region bounded by $x^2 + y^2 = 9,\ z = 0,\ z = 4$.

Step 1. Is the region closed?

Step 2. Is the integrand a vector-valued function?

Step 3. Identify $\mathbf{F}$.

Step 4. Compute div $\mathbf{F}$.

Step 5. Evaluate $\iiint_E \text{div } \mathbf{F} dV$.

Problem 3. Evaluate $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} - yz\mathbf{k}$. E is the solid region bounded by $z = x^2 + y^2$ and $z = 4$.

Step 1. Is the region closed?

Step 2. Is the integrand a vector-valued function?

Step 3. Identify $\mathbf{F}$.

Step 4. Compute div $\mathbf{F}$.

Step 5. Evaluate $\iiint_E \text{div } \mathbf{F} dV$.

Problem 4. Evaluate $\mathbf{F} = (xy^2 + \cos z)\mathbf{i} + 9x^2y + \sin z)\mathbf{j} + e^z\mathbf{k}$. E is the solid region bounded by the cone $z = \sqrt{x^2 + y^2}$ and $z = 4$.

Step 1. Is the region closed?

Step 2. Is the integrand a vector-valued function?

Step 3. Identify $\mathbf{F}$.

Step 4. Compute div $\mathbf{F}$.

Step 5. Evaluate $\iiint_E \text{div } \mathbf{F} dV$.