Exam No. 01 (Fall 2013)

PHYS 320: Electricity and Magnetism I

Date: 2013 Sep 18

1. (**20 points.**) The relation between the vector potential $\mathbf{A}$ and the magnetic field $\mathbf{B}$ is

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{1}$$

For a constant (homogeneous in space) magnetic field $\mathbf{B}$, verify that

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r} \tag{2}$$

is a possible vector potential by showing that Eq. (2) satisfies Eq. (1).

2. (**20 points.**) A gyroid is an (infinitely connected triply periodic minimal) surface discovered by Alan Schoen in 1970. Schoen presently resides in Carbondale and was a professor at SIU in the later part of his career. Apparently, a gyroid is approximately described by the surface

$$f(x, y, z) = \cos x \sin y + \cos y \sin z + \cos z \sin x \tag{3}$$

when $f(x, y, z) = 0$. Using the fact that the gradient operator

$$\boldsymbol{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z} \tag{4}$$

determines the normal vectors on a surface, evaluate

$$\boldsymbol{\nabla} f(x, y, z). \tag{5}$$

3. (**20 points.**) Evaluate the integral

$$\int_{-1}^{1} \frac{\delta(1 - 3x)}{x}\, dx. \tag{6}$$

Hint: Be careful to avoid a possible error in sign.

4. (**20 points.**) Evaluate the vector area of a hemispherical bowl of radius R given by

$$\mathbf{a} = \int_S d\mathbf{a}. \tag{7}$$

5. (**20 points.**) Evaluate the number evaluated by the expression

$$\frac{1}{2}\left[\hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial}{\partial\phi}\right]\cdot(\rho\hat{\boldsymbol{\rho}}), \tag{8}$$

where $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\phi}}$ are the unit vectors for cylindrical coordinates (ρ, ϕ) given by

$$\hat{\boldsymbol{\rho}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}}, \tag{9}$$

$$\hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}. \tag{10}$$