Homework No. 01 (Fall 2013)

PHYS 320: Electricity and Magnetism I

Due date: Wednesday, 2013 Aug 28, 4.30pm

1. Problem 1.2, Griffiths 4th edition.
2. Find the angle between the face diagonal (1,0,1) and the body diagonal (1,1,1) of the cube in Figure 1.10 of Griffiths 4th edition. (Modified version of Example 1.2, Griffiths 4th edition.)
3. Problem 1.4, Griffiths 4th edition.
4. Using antisymmetric property of Levi-Civita symbol show that
$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = -\mathbf{A}\cdot\mathbf{C}\times\mathbf{B}. \tag{1}$$
5. Spherical polar coordinates are coordinated by the intersection of family of spheres, cones, and half-planes, given by
$$r = \sqrt{x^2+y^2+z^2}, \tag{2a}$$
$$\theta = \tan^{-1}\sqrt{\frac{x^2+y^2}{z^2}}, \tag{2b}$$
$$\phi = \tan^{-1}\frac{y}{x}, \tag{2c}$$
respectively. Show that the gradient of these surfaces are given by
$$\boldsymbol{\nabla} r = \hat{\mathbf{r}}, \qquad \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}, \tag{3a}$$
$$\boldsymbol{\nabla}\theta = \hat{\boldsymbol{\theta}}\frac{1}{r}, \qquad \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{i}} + \cos\theta\sin\phi\,\hat{\mathbf{j}} - \sin\theta\,\hat{\mathbf{k}}, \tag{3b}$$
$$\boldsymbol{\nabla}\phi = \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}, \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}, \tag{3c}$$
which are normal to the respective surfaces. Sketch the surfaces and the corresponding normal vectors on a diagram.
6. Show that
   (a) $\boldsymbol{\nabla} r = \hat{\mathbf{r}}$
   (b) $\boldsymbol{\nabla}\,\mathbf{r} = \mathbf{1}$
   (c) $\boldsymbol{\nabla}\cdot\mathbf{r} = 3$

(d) $\boldsymbol{\nabla} \times \mathbf{r} = 0$

7. Show that

   (a) $\boldsymbol{\nabla} \frac{\mathbf{r}}{r^n} = \mathbf{1}\frac{1}{r^n} - n\frac{\mathbf{r}\,\mathbf{r}}{r^{n+2}}$

   (b) $\boldsymbol{\nabla} \cdot \frac{\mathbf{r}}{r^n} = \frac{(3-n)}{r^n}$

   (c) $\boldsymbol{\nabla} \times \frac{\mathbf{r}}{r^n} = 0$

   (d) Comment on Problem 1.16, Griffiths 4th edition.

8. Show that

   (a) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

   (b) $\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = (\boldsymbol{\nabla} \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B})$