# Exam No. 03 (Fall 2013)

## PHYS 520A: Electromagnetic Theory I

Date: 2013 Nov 21

1. (**30 points.**) A uniformly polarized sphere of radius $R$ is described by

$$\mathbf{P}(\mathbf{r}) = \alpha r^2 \,\hat{\mathbf{r}}\, \theta(R - r). \tag{1}$$

Find the effective charge density by calculating $-\boldsymbol{\nabla} \cdot \mathbf{P}$. In particular, you should obtain two terms, one containing $\theta(R - r)$ that is interpreted as a volume charge density, and another containing $\delta(R - r)$ that can be interpreted as a surface charge density.

2. (**30 points.**) Consider the Green's function equation

$$-\left(\frac{d^2}{dt^2} + \omega^2\right) G(t) = \delta(t). \tag{2}$$

Verify, by substituting into Eq. (2), that

$$G(t) = -\frac{1}{\omega}\theta(t) \sin \omega(t), \tag{3}$$

is a particular solution to the Green's function equation.

3. (**40 points.**) The expression for the electric potential due to a point charge placed in between two perfectly conducting semi-infinite slabs described by

$$\varepsilon(z) = \begin{cases} \infty, & z < 0, \\ \varepsilon_0, & 0 < z < a, \\ \infty, & a < z, \end{cases} \tag{4}$$

is given in terms of the reduced Green's function that satisfies the differential equation $(0 < \{z, z'\} < a)$

$$\left[-\frac{\partial^2}{\partial z^2} + k^2\right] \varepsilon_0 g(z, z') = \delta(z - z') \tag{5}$$

with boundary conditions requiring the reduced Green's function to vanish at $z = 0$ and $z = a$.

(a) Construct the reduced Green's function in the form

$$g(z, z') = \begin{cases} A \sinh kz + B \cosh kz, & 0 < z < z' < a, \\ C \sinh kz + D \cosh kz, & 0 < z' < z < a, \end{cases} \tag{6}$$

and solve for the four coefficients, $A, B, C, D$, using the conditions

$$g(0, z') = 0, \tag{7a}$$

$$g(a, z') = 0, \tag{7b}$$

$$g(z, z')\Big|_{z=z'-\delta}^{z=z'+\delta} = 0, \tag{7c}$$

$$\varepsilon_0 \partial_z g(z, z')\Big|_{z=z'-\delta}^{z=z'+\delta} = -1. \tag{7d}$$

Hint: The hyperbolic functions here are defined as

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \text{and} \qquad \cosh x = \frac{1}{2}(e^x + e^{-x}). \tag{8}$$

(b) Take the limit $ka \to \infty$ in your solution above to obtain the reduced Green's function for a single perfectly conducting slab,

$$\lim_{ka\to\infty} g(z, z') = \frac{1}{\varepsilon_0}\frac{1}{2k}e^{-k|z-z'|} - \frac{1}{\varepsilon_0}\frac{1}{2k}e^{-k|z|}e^{-k|z'|}. \tag{9}$$

This should serve as a check for your solution to the reduced Green's function.