Final Exam (Fall 2013)

PHYS 530B: Quantum Mechanics II

Date: 2013 Dec 9

1. (**25 points.**) Construct the total angular momentum state $|N, N\rangle$ for the composite system built out of two angular momenta $j_1 = N$ and $j_2 = 1$.

2. (**25 points.**) The components of the position and momentum operator, $\mathbf{r}$ and $\mathbf{p}$, respectively, satisfy the commutation relations $[r_i, p_j] = i\hbar\delta_{ij}$. Evaluate

$$r^2\,\mathbf{p} - \mathbf{p}\,r^2. \tag{1}$$

3. (**25 points.**) Verify, by substitution, that

$$G(\mathbf{r}) = -\frac{e^{ikr}}{4\pi r} \tag{2}$$

is a particular solution to the Green's function equation

$$\left[\nabla^2 + k^2\right] G(\mathbf{r}) = \delta^{(3)}(\mathbf{r}). \tag{3}$$

4. (**25 points.**) The Dirac equation is described by the Hamiltonian

$$H = \boldsymbol{\alpha} c \cdot \mathbf{p} + \beta mc^2, \tag{4}$$

where $\boldsymbol{\alpha}$ and β are anticommuting operators that satisfy

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}, \qquad \alpha_i\beta + \beta\alpha_i = 0, \qquad \beta^2 = 1. \tag{5}$$

Evaluate the commutation relation $[\mathbf{r}, H]$. Thus, derive the equation of motion

$$\frac{d\mathbf{r}}{dt} = \frac{1}{i\hbar}\,[\mathbf{r}, H] = ?. \tag{6}$$

Comparing the above relativistic equation of motion with the corresponding non-relativistic equation of motion give a plausible physical interpretation for the operator $\boldsymbol{\alpha} c$.