Homework No. 01 (Fall 2013)

PHYS 530B: Quantum Mechanics II

Due date: Friday, 2013 Aug 30, 4.30pm

1. (Ref: Milton's notes.) The energy of a charge e moving with velocity $\mathbf{v}$ in an external electromagnetic field is
$$E = e\phi - \frac{e}{c}\mathbf{v}\cdot\mathbf{A}, \tag{1}$$
where ϕ is the scalar potential and $\mathbf{A}$ is the vector potential. The relation between $\mathbf{A}$ and the magnetic field $\mathbf{H}$ is
$$\mathbf{H} = \boldsymbol{\nabla}\times\mathbf{A}. \tag{2}$$
For a constant (homogenous in space) magnetic field $\mathbf{H}$, verify that
$$\mathbf{A} = \frac{1}{2}\mathbf{H}\times\mathbf{r} \tag{3}$$
is a possible vector potential. Then, by looking at the energy, identify the magnetic moment $\boldsymbol{\mu}$ of the moving charge.

2. (Ref: Milton's notes.) Consider an atom entering a Stern-Gerlach apparatus. Deflection upward begins as soon as the atom enters the inhomogeneous field. By the time the atom leaves the field, it has been deflected upward by a net amount Δz. Compute Δz for
$$\mu_z = 10^{-27}\frac{\mathrm{J}}{\mathrm{G}}, \quad \frac{\partial H_z}{\partial z} = 10^6\frac{\mathrm{G}}{\mathrm{m}}, \quad l = 10\,\mathrm{cm}, \quad T = \frac{mv_x^2}{k} = 10^3\,\mathrm{K}. \tag{4}$$

3. (Ref: Milton's notes.) A silver atom has mass (actually the stable isotopes are Ag^{107}, Ag^{109})
$$m = 108\times 1.67\times 10^{-27}\,\mathrm{kg}, \tag{5}$$
and speed
$$v = 10^2\,\mathrm{m/s}. \tag{6}$$
Compute the reduced de Broglie wavelength, $\lambda\!\!\!^{-}$, and the corresponding diffraction angle $\delta\theta$ when a beam of such atoms passes through a slit of width $10^{-2}\,\mathrm{cm}$. (See Fig. 3.3 in Milton's notes and discussion of Eq. (3.26) there.) Compare this diffraction angle with the deflection angle produced in a Stern-Gerlach experiment.

4. (Ref: Milton's notes.) Using the notation for the probability for a measurement in the Stern-Gerlach experiment, introduced in the class, show that
$$p\big([+;\theta_1,\phi_1]\to[-;\theta_2,\phi_2]\big) = \frac{1-\cos\Theta}{2}, \tag{7}$$
where
$$\cos\Theta = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2). \tag{8}$$

5. Show that

$$p\big([+;0,0] \to [+;\pi,0]\big) = 0. \tag{9}$$

Further, show that

$$p\big([+;0,0] \to [\pm;\theta,\phi] \to [+;\pi,0]\big) = 0, \tag{10}$$

which is a statement of destructive interference. Compare this with the probability for

$$p\big([+;0,0] \to [+;\theta,\phi] \to [+;\pi,0]\big) \tag{11}$$

and

$$p\big([+;0,0] \to [-;\theta,\phi] \to [+;\pi,0]\big). \tag{12}$$

6. Show that

$$p\big([+;0,0] \to [-;\pi,0]\big) = 1. \tag{13}$$

Further, show that

$$p\big([+;0,0] \to [\pm;\theta,\phi] \to [-;\pi,0]\big) = 1, \tag{14}$$

which is a statement of constructive interference. Compare this with the probability for

$$p\big([+;0,0] \to [+;\theta,\phi] \to [-;\pi,0]\big) \tag{15}$$

and

$$p\big([+;0,0] \to [-;\theta,\phi] \to [-;\pi,0]\big). \tag{16}$$