Homework No. 03 (Fall 2013)

PHYS 530B: Quantum Mechanics II

Due date: Wednesday, 2013 Sep 25, 4.30pm

1. For $j = 1$:

   (a) Determine the matrix representantion for
   $$J_z, J_x, J_y, J_+, J_-, \text{and } J^2. \tag{1}$$

   (b) Evaluate
   $$\mathrm{Tr}(J_k), \quad \mathrm{Tr}(J_k J_l), \quad \text{and} \quad \mathrm{Tr}(J_k^2 J_l^2), \quad \text{for} \quad k, l = x, y, z. \tag{2}$$

2. Using the asymptotic form for Hermite polynomials, $H_n(x)$, for large n, discuss the manner in which the harmonic oscillator eigenfunctions approach those of the free particle in the limit when the frequency of oscillations $\omega \to 0$.

3. (Set $\hbar = 1$.) From
   $$y|n\rangle = \sqrt{n}|n-1\rangle \tag{3}$$
   derive
   $$\frac{d}{dx} H_n(x) = 2n H_{n-1}(x). \tag{4}$$
   Check this for $n = 4, 3, 2, 1, 0$. From
   $$y^\dagger |n\rangle = \sqrt{n+1}|n+1\rangle \tag{5}$$
   derive
   $$\left(2x - \frac{d}{dx}\right) H_n(x) = H_{n+1}(x). \tag{6}$$
   Add the two statements to obtain
   $$2x H_n(x) = H_{n+1}(x) + 2n H_{n-1}(x). \tag{7}$$
   This recursion relation gives a way of recursively calculating $H_{n+1}(x)$ in terms of $H_n(x)$ and $H_{n-1}(x)$. Check this for $n = 3, 2, 1, 0$.

4. Use the results of Problem 3 to deduce the differential equation
   $$\left(\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 2n\right) H_n(x) = 0. \tag{8}$$
   Show the equivalence of this with
   $$\left(\frac{d^2}{dx^2} - x^2 + 2n + 1\right) \psi_n(x) = 0. \tag{9}$$
   This is the "time-independent Schrödinger equation" for the harmonic oscillator.