Take-Home Exam No. 01 (Spring 2014)

PHYS 420: Electricity and Magnetism II

Due date: 2014 Mar 17

1. An infinitely long wire of circular cross section radius a carries a steady current I. Another wire, in the form of a cylindrical shell and concentric to the first wire, has inner radius b and outer radius c, such that $a < b < c$. The region enclosed by $a < \rho < b$ and $c < \rho$ is empty space. The outer wire carries the same current I in the opposite direction. Let the direction of z-axis be along the wire.

   (a) (**40 points.**) Use Ampere's law to find the expression for magnetic field in the four regions, $\rho < a$, $a < \rho < b$, $b < \rho < c$, and $c < \rho$.

   (b) (**20 points.**) Plot the resulting magnetic field as a function of ρ.

2. (**40 points.**) Generate 3D plots of surface spherical harmonics $Y_{lm}(\theta, \phi)$ as a function of θ and ϕ. In particular,

   (a) Plot $\mathrm{Re}\big[Y_{73}(\theta, \phi)\big]$.

   (b) Plot $\mathrm{Im}\big[Y_{73}(\theta, \phi)\big]$.

   (c) Plot $\mathrm{Abs}\big[Y_{73}(\theta, \phi)\big]$.

   Hint: In Mathematica these plots are generated using the following commands:

```
SphericalPlot3D[Re[SphericalHarmonicY[l, m, θ, ϕ]], {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
SphericalPlot3D[Im[SphericalHarmonicY[l, m, θ, ϕ]], {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
SphericalPlot3D[Abs[SphericalHarmonicY[l, m, θ, ϕ]], {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
```