Homework No. 03 (Spring 2014)

PHYS 420: Electricity and Magnetism II

Due date: Tuesday, 2014 Feb 18, 4.30pm

1. Problem 5.41, Griffiths 4th edition. This problem is based on the phenomenon called the Hall effect.

2. A way of determining the sign of charge carriers in a conductor is by means of the Hall effect. A magnetic field $\mathbf{B}$ is applied perpendicular to the direction of current flow in a conductor, and as a consequence a transverse voltage drop appears across the conductor. If d is the transverse length of the conductor, and v is the average drift speed of the charge carriers, show that the voltage, in magnitude, is

$$V = vBd. \tag{1}$$

   Estimate this potential drop (magnitude and direction) for a car driving towards North in the Northern hemisphere. How will the answer differ in the Southern hemisphere?

3. (Based on Problem 5.8, Griffiths 4th edition.) The magnetic field at position $\mathbf{r} = (x, y, z)$ due to a finite wire segment of length $2L$ carrying a steady current I, with the caveat that it is unrealistic (why?), placed on the z-axis with its end points at $(0, 0, L)$ and $(0, 0, -L)$, is

$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\, \frac{\mu_0 I}{4\pi} \frac{1}{\sqrt{x^2 + y^2}} \left[\frac{z + L}{\sqrt{x^2 + y^2 + (z + L)^2}} - \frac{z - L}{\sqrt{x^2 + y^2 + (z - L)^2}} \right], \tag{2}$$

   where $\hat{\boldsymbol{\phi}} = (-\sin\phi\, \hat{\mathbf{i}} + \cos\phi\, \hat{\mathbf{j}}) = (-y\, \hat{\mathbf{i}} + x\, \hat{\mathbf{j}})/\sqrt{x^2 + y^2}$.

   (a) Show that by taking the limit $L \to \infty$ we obtain the magnetic field near a long straight wire carrying a steady current I,

$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\, \frac{\mu_0 I}{2\pi\rho}, \tag{3}$$

   where $\rho = \sqrt{x^2 + y^2}$ is the perpendicular distance from the wire.

   (b) Show that the magnetic field on a line bisecting the wire segment is given by

$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\, \frac{\mu_0 I}{2\pi\rho} \frac{L}{\sqrt{\rho^2 + L^2}}. \tag{4}$$

(c) Find the magnetic field at the center of a square loop, which carries a steady current I. Let $2L$ be the length of a side, ρ be the distance from center to side, and $R = \sqrt{\rho^2 + L^2}$ be the distance from center to a corner. (Caution: Notation differs from Griffiths.) You should obtain

$$B = \frac{\mu_0 I}{2R} \frac{4}{\pi} \tan \frac{\pi}{4}. \tag{5}$$

(d) Show that the magnetic field at the center of a regular n-sided polygon, carrying a steady current I is

$$B = \frac{\mu_0 I}{2R} \frac{n}{\pi} \tan \frac{\pi}{n}, \tag{6}$$

where R is the distance from center to a corner of the polygon.

(e) Show that the magnetic field at the center of a circular loop of radius R,

$$B = \frac{\mu_0 I}{2R}, \tag{7}$$

is obtained in the limit $n \to \infty$.