Homework No. 07 (Spring 2014)

PHYS 420: Electricity and Magnetism II

Due date: Not applicable

1. Consider the motion of a non-relativisitic particle (speed v small compared to speed of light c, $v \ll c$,) of charge q and mass m. The charge oscillates on the x-axis with frequency ω_0 and amplitude A given by

$$\mathbf{r}_a(t) = \hat{\mathbf{i}}\, A \cos \omega_0 t. \tag{1}$$

   (a) Find the acceleration of the particle

$$\mathbf{a}_a(t) = \frac{d^2}{dt^2}\mathbf{r}_a(t). \tag{2}$$

   (b) Find the angular distribution of the radiated power

$$f(\theta, \phi, t) = \frac{dP}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4\pi c^3}\big[\hat{\mathbf{r}} \times \mathbf{a}(t_e)\big]^2 \tag{3}$$

   and the total radiated power

$$P(t) = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3c^3}\mathbf{a}^2(t_e), \tag{4}$$

   where $\mathbf{a}(t_e)$ is the acceleration of the particle at the time of emission

$$t_e = t - \frac{r}{c}. \tag{5}$$

2. Consider the motion of two non-relativisitic particles (speed v_i small compared to speed of light c, $v_i \ll c$,) of identical charges $q_i = q$ and identical masses $m_i = m$, $i = 1, 2$. The individual radiation fields $\mathbf{B}_i(\mathbf{r}, t)$ and $\mathbf{E}_i(\mathbf{r}, t)$, the angular distribution of emitted power $f_i(\theta, \phi, t)$, and the total radiated power $P_i(t)$, are given by the expressions,

$$c\mathbf{B}_i(\mathbf{r}, t) = -\frac{\mu_0}{4\pi}\frac{q}{r}\,\hat{\mathbf{r}} \times \mathbf{a}_i(t_e), \tag{6a}$$

$$\mathbf{E}_i(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\frac{q}{r}\,\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}_i(t_e)), \tag{6b}$$

$$f_i(\theta, \phi, t) = \frac{dP_i}{d\Omega} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4\pi c^3}\big[\hat{\mathbf{r}} \times \mathbf{a}_i(t_e)\big]^2, \tag{6c}$$

$$P_i(t) = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3c^3}\mathbf{a}_i^2(t_e), \tag{6d}$$

where $\mathbf{a}_i(t_e)$ is the acceleration of the i-th particle at the time of emission

$$t_e = t - \frac{r}{c}. \tag{7}$$

Let the total contribution to a physical quantity from the two particles together be denoted by the subscript $(1+2)$.

(a) Show that

$$\mathbf{B}_{(1+2)}(\mathbf{r},t) = \mathbf{B}_1(\mathbf{r},t) + \mathbf{B}_2(\mathbf{r},t), \tag{8a}$$

$$\mathbf{E}_{(1+2)}(\mathbf{r},t) = \mathbf{E}_1(\mathbf{r},t) + \mathbf{E}_2(\mathbf{r},t), \tag{8b}$$

thus, conclude that radiation fields are additive.

(b) Show that, in general, the angular distribution of radiated power and total radiated power from the two particles together is not additive and exhibits interference,

$$f_{(1+2)}(\theta,\phi,t) = f_1(\theta,\phi,t) + f_2(\theta,\phi,t) + f_{12}(\theta,\phi,t), \tag{9}$$

where

$$f_{12}(\theta,\phi,t) = 2\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4\pi c^3}\big[\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e) - (\hat{\mathbf{r}}\cdot\mathbf{a}_1(t_e))(\hat{\mathbf{r}}\cdot\mathbf{a}_2(t_e))\big], \tag{10}$$

and

$$P_{(1+2)}(t) = P_1(t) + P_2(t) + P_{12}(t), \tag{11}$$

where

$$P_{12}(t) = 2\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3c^3}\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e). \tag{12}$$

Observe that the interference effect in the total radiated power is totally destructive for the case $\mathbf{a}_1(t_e)\cdot\mathbf{a}_2(t_e) = 0$. For this case, the interference effect in the angular distribution of radiated power is not necessarily destructive.

(c) Consider the motion of two particles moving on a circle with same uniform speed while remaining diametrically opposite to each other at each moment. Find the total radiated power $P_{(1+2)}(t)$. (Hint: The centripetal acceleration is in the radial direction.)

(d) Consider the motion of three particles moving on a circle with same uniform speed while remaining at the vertices of a equilateral triangle at each moment. Find the total radiated power $P_{(1+2+3)}(t)$.

(e) Find $P_{(1+2+3+4)}(t)$ for four particles moving on a circle such that they are at the vertices of a square at each moment.

(f) Find $P_{(1+\ldots+N)}(t)$ for N particles moving on a circle such that they are at the vertices of a N-sided polygon at each moment.