Midterm Exam No. 02 (Spring 2014)

PHYS 520B: Electromagnetic Theory

Date: 2014 Apr 7

1. (**20 points.**) The magnetic field at a distance R from a wire of infinite extent carrying a steady current I is given by
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{\boldsymbol{\phi}}, \tag{1}$$
where the direction of $\hat{\boldsymbol{\phi}}$ is given by the right-hand rule. Find the magnetic field at point o in Fig. 1 in terms of distances a and b and current I.

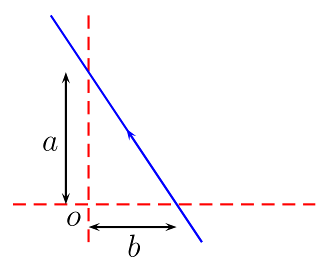

Figure 1: Problem 1

2. (**20 points.**) From Maxwell's equations, including magnetic charges and currents,
$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{1}{\varepsilon_0}\rho_e, \tag{2a}$$
$$\boldsymbol{\nabla}\cdot\mathbf{H} = \frac{1}{\mu_0}\rho_m, \tag{2b}$$
$$-\boldsymbol{\nabla}\times\mathbf{E} - \mu_0\frac{\partial\mathbf{H}}{\partial t} = \mathbf{J}_m, \tag{3a}$$
$$\boldsymbol{\nabla}\times\mathbf{H} - \varepsilon_0\frac{\partial\mathbf{E}}{\partial t} = \mathbf{J}_e, \tag{3b}$$
derive the inhomogeneous wave equation
$$\left(-\nabla^2 + \varepsilon_0\mu_0\frac{\partial^2}{\partial t^2}\right)\mathbf{H} = -\frac{1}{\mu_0}\boldsymbol{\nabla}\rho_m - \varepsilon_0\frac{\partial}{\partial t}\mathbf{J}_m + \boldsymbol{\nabla}\times\mathbf{J}_e. \tag{4a}$$

3. (**20 points.**) Using the identity
$$\delta(F(x)) = \sum_r \frac{\delta(x-a_r)}{\left|\frac{dF}{dx}\big|_{x=a_r}\right|}, \tag{5}$$
where the sum on r runs over the roots a_r of the equation $F(x)=0$, evaluate
$$\delta(x^3 - 6x^2 + 11x - 6). \tag{6}$$

4. (**20 points.**) The 4-dimensional Euclidean Green's function satisfies

$$-\left(\boldsymbol{\nabla}^2 + \frac{\partial^2}{\partial x_4^2}\right) G_E(\mathbf{r}, x_4) = \delta^{(3)}(\mathbf{r})\delta(x_4) \tag{7}$$

and has the solution

$$G_E(\mathbf{r}, x_4) = \frac{1}{4\pi^2}\frac{1}{\mathbf{r}^2 + x_4^2}. \tag{8}$$

Evaluate the integral

$$\int_{-\infty}^{\infty} dx_4\, G_E(\mathbf{r}, x_4). \tag{9}$$

From the answer what can you comment about the physical interpretation of $\int_{-\infty}^{\infty} dx_4\, G_E$?

5. (**20 points.**) Consider a point electric dipole moment $\mathbf{d}$ moving with velocity $\mathbf{v} = v\hat{\mathbf{z}}$. For the case of time independent $\mathbf{d}$ and $\mathbf{v}$, and when the dipole moves close to speed of light, $\beta \to 1$, we can write the leading order contributions in $(1-\beta^2)$ for the electric and magnetic fields as

$$\mathbf{E}(\mathbf{r}, t) = \begin{cases} \dfrac{1}{\sqrt{1-\beta^2}}\dfrac{1}{4\pi\varepsilon_0}(-\mathbf{d}\cdot\boldsymbol{\nabla})\dfrac{\boldsymbol{\rho}}{\rho^3}, & z = vt, \\ 0, & z \neq vt, \end{cases} \tag{10a}$$

$$c\mathbf{B}(\mathbf{r}, t) = \begin{cases} \dfrac{\beta}{\sqrt{1-\beta^2}}\dfrac{1}{4\pi\varepsilon_0}(-\mathbf{d}\cdot\boldsymbol{\nabla})\dfrac{\boldsymbol{\phi}}{\rho^3}, & z = vt, \\ 0, & z \neq vt, \end{cases} \tag{10b}$$

where $\boldsymbol{\rho} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ and $\boldsymbol{\phi} = -y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$. These fields are confined on a plane perpendicular to direction of motion. Determine the electromagnetic momentum density flux for the particular configuration $\mathbf{d} = d\hat{\boldsymbol{\rho}}$ by calculating

$$\mathbf{E}\times\mathbf{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,\mathbf{E}\times c\mathbf{B}. \tag{11}$$