Take-Home Exam No. 01 (Spring 2014)

PHYS 520B: Electromagnetic Theory

Due date: 2014 Mar 17

The vector potential inside a rotating charged conducting shell is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{Q}{3R}\,\boldsymbol{\omega}_0 \times \mathbf{r}, \qquad r < R, \tag{1}$$

where Q is the total charge on the conducting shell of radius R that is rotating with angular velocity $\boldsymbol{\omega}_0$.

1. (**10 points.**) Show that the magnetic field produced by this motion is given

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} = \frac{\mu_0}{4\pi}\frac{2Q}{3R}\,\boldsymbol{\omega}_0, \qquad r < R, \tag{2}$$

   which is uniform inside the shell.

2. (**20 points.**) A charged particle takes a circular path (in general a helical path). Describe the motion of an electron inside this rotating shell. In particular, calculate the expression for the angular speed of rotation ω of the electron.

3. (**20 points.**) Next, consider a current carrying loop of wire inside the shell. The interaction energy of this loop with the rotating shell is given by

$$W_m = -\int d^3r \mathbf{J}(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r}), \tag{3}$$

   where $\mathbf{J}(\mathbf{r})$ is the current density of the current carrying loop. Show that this interaction energy can be expressed in terms of the magnetic field as

$$W_m = -I \int_S d\mathbf{S} \cdot \mathbf{B}(\mathbf{r}) = -I\boldsymbol{\Phi}_m, \tag{4}$$

   where I is the current in the loop, S represents any surface bounded by the loop, and $\boldsymbol{\Phi}_m$ is the magnetic flux through the loop.

4. (**20 points.**) Calculate the interaction energy between a circular loop of wire of radius a carrying a current I with the symmetrical axis of the loop along the direction $\mathbf{n}$.

5. (**20 points.**) Torque is defined as negative change in energy with respect to a change in rotation angle θ,

$$\tau = -\frac{\partial}{\partial\theta} W_m. \tag{5}$$

   Define $\cos\theta = \mathbf{n} \cdot \hat{\boldsymbol{\omega}}_0$ and calculate the torque on the current loop inside the shell.

6. (**10 points.**) Precession of a spinning top is understood in terms of the torque equation. Do you expect a current loop inside the rotating shell to precess? Or, in general, a particle with magnetic moment to precess inside the rotating shell?