Homework No. 06 (Spring 2014)

PHYS 520B: Electromagnetic Theory

Due date: Thursday, 2014 Apr 24, 4.30pm

1. Show that
$$\boldsymbol{\nabla}_r(\hat{\mathbf{r}}\cdot\mathbf{r}') = -\frac{1}{r}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{r}'). \tag{1}$$

2. A particle, of charge q and mass m, always moves with speed $v \ll c$.

   (a) Consider the case when it oscillates on the x-axis with frequency ω_0 and amplitude A given by
   $$\mathbf{r}_1(t) = \hat{\mathbf{x}}A\cos\omega_0 t. \tag{2}$$
   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r},t)$, radiated magnetic field $\mathbf{B}(\mathbf{r},t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (b) Next, consider the case when the particle moves on a circle described by
   $$\mathbf{r}_2(t) = \hat{\mathbf{x}}A\cos\omega_0 t + \hat{\mathbf{y}}A\sin\omega_0 t. \tag{3}$$
   Obtain expressions for the radiated electric field $\mathbf{E}(\mathbf{r},t)$, radiated magnetic field $\mathbf{B}(\mathbf{r},t)$, angular distribution of the radiated power $dP/d\Omega$, and the total power radiated P.

   (c) Show that the radiated electric and magnetic field is additive, that is, it is the sum of two oscillators.

   (d) Show that the radiated power is not additive, but exhibits interference effects. Identify the interference term for the circular motion.

   (e) Find directions $\hat{\mathbf{r}}$ for which the interference term goes to zero.

3. (Schwinger et al., problem 32.1.) A particle, of charge q and mass m, moves with speed $v \ll c$, in a uniform magnetic field $\mathbf{B}$. Suppose the motion is confined to the plane perpendicular to $\mathbf{B}$. Calculate the power radiated P in terms of B and v, and show that
$$P = -\frac{dE}{dt} = \gamma E, \tag{4}$$
where $E = mv^2/2$ is the energy of the particle. Find γ. Since then
$$E(t) = E(0)\,e^{-\gamma t}, \tag{5}$$
$1/\gamma$ is the mean lifetime of the motion. For an electron, find $1/\gamma$ in seconds for a magnetic field of 10^4 Gauss.

4. (Schwinger et al., problem 32.2.) A nonrelativistic particle of charge q and mass m moves in a Hooke's law potential (a linear oscillator) with natural frequency ω_0. Find P, the power radiated. Recall that for such motion, the time-averaged kinetic and potential energy satisfy

$$\bar{T} = \bar{V} = \frac{1}{2}E. \tag{6}$$

Show then that the power radiated, averaged over one cycle is

$$P = -\frac{dE}{dt} = \gamma E, \tag{7}$$

and find γ. Compute $1/\gamma$ in seconds, for electron, when ω_0 is $10^{15}\,\mathrm{sec}^{-1}$ (a characteristic atomic frequency, corresponding to visible light).