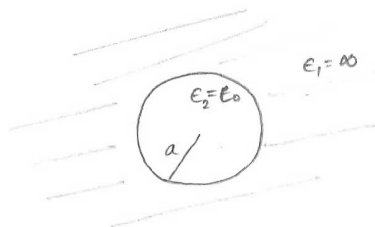


Green's function for a perfectly conducting cylinder (inside)

①



② Our goal is to solve the differential equation

$$-\epsilon_0 \nabla^2 G(\vec{r}, \vec{r}') = \delta^{(3)}(\vec{r} - \vec{r}')$$

$$③ \quad G(\vec{r}, \vec{r}') = \frac{1}{\epsilon_0} \int_{-\infty}^{+\infty} \frac{dk_z}{2\pi} e^{ik_z(z-z')} \sum_{m=-\infty}^{+\infty} \frac{1}{2\pi i} e^{im(\phi-\phi')} g_m(\rho, \rho'; k_z)$$

④ Substituting ③ in ② we obtain the differential

$$\text{equation}$$

$$\left[-\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{m^2}{\rho^2} + k_z^2 \right] g_m(\rho, \rho'; k_z) = \frac{\delta(\rho - \rho')}{\rho}$$

⑤ Let us construct

$$g_m(\rho, \rho'; k_z) = \begin{cases} A I_m(k_z \rho) + B K_m(k_z \rho) & \rho < \rho' \\ C I_m(k_z \rho) + D K_m(k_z \rho) & \rho' < \rho \end{cases}$$

⑥ We immediately see that

$$B = 0,$$

because otherwise the solution will blow up at the origin.

⑦ We have the conditions

$$(i) \quad g_m(r, r'; k_2) \Big|_{r=r'+\delta}^{r=r'-\delta} = 0$$

$$(ii) \quad r \frac{\partial}{\partial r} g_m(r, r'; k_2) \Big|_{r=r'+\delta}^{r=r'-\delta} = -1$$

$$(iii) \quad g_m(a, r'; k_2) = 0 \quad \longrightarrow \quad \text{property of a perfect conductor.}$$

⑧ Using ⑦ - (iii) we have.

$$C I_m(k_2 a) + D K_m(k_2 a) = 0$$

$$I_a \equiv I_m(k_2 a)$$

$$K_a \equiv K_m(k_2 a)$$

$$D = -C \frac{I_a}{K_a}.$$

⑨ Using ⑧ and ⑥ in ⑤

$$g_m(s, s'; k_2) = \begin{cases} A I_m(k_2 s) & s < s' \\ C I_m(k_2 s) - C \frac{I_a}{K_a} K_m(k_2 s) & s' < s \end{cases}$$

⑩ Using ⑨ in ⑦-(i) and ⑦-(ii)

$$\begin{aligned} K_m'(k_2 s') \times & \left\{ \begin{aligned} & C I_m(k_2 s') - C \frac{I_a}{K_a} K_m(k_2 s') - A I_m(k_2 s') = 0 \quad \text{--- (i)} \\ & C I_m'(k_2 s') - C \frac{I_a}{K_a} K_m'(k_2 s') - A I_m'(k_2 s') = -\frac{1}{k_2 s'} \quad \text{--- (ii)} \end{aligned} \right. \\ K_m(k_2 s') \times & \end{aligned}$$

⑪ Subtracting

$$(C - A) \left[\underbrace{I_m(k_2 s') K_m'(k_2 s') - K_m(k_2 s') I_m'(k_2 s')}_{\text{Wronskian} = -\frac{1}{k_2 s'}} \right] = \frac{1}{k_2 s'} K_m(k_2 s')$$

$$A - C = K_m(k_2 s')$$

⑫ Using ⑪ in ⑩-(i)

$$\begin{aligned} \checkmark C I_m(k_2 s') - C \frac{I_a}{K_a} K_m(k_2 s') - \left(\checkmark C + K_m(k_2 s') \right) I_m(k_2 s') &= 0 \\ - C \frac{I_a}{K_a} K_m(k_2 s') &= K_m(k_2 s') I_m(k_2 s') \\ C &= - \frac{K_a}{I_a} I_m(k_2 s') \end{aligned}$$

(13) Using (11) and (12)

$$C = - \frac{K_a}{I_a} I_m(k_2 s')$$

$$A = K_m(k_2 s') - \frac{K_a}{I_a} I_m(k_2 s')$$

(14) Using (13) in (9)

$$g_m(s, s'; k_2) = \begin{cases} I_m(k_2 s') K_m(k_2 s) - \frac{K_a}{I_a} I_m(k_2 s) I_m(k_2 s') & s < s' \\ I_m(k_2 s') K_m(k_2 s) - \frac{K_a}{I_a} I_m(k_2 s) I_m(k_2 s') & s' < s \end{cases}$$

$$= I_m(k_2 s') K_m(k_2 s) - \frac{K_a}{I_a} I_m(k_2 s) I_m(k_2 s')$$