Midterm Exam No. 01 (Spring 2014)

PHYS 530A: Quantum Mechanics II

Date: 2014 Feb 13

1. (**20 points.**) The projection operator is defined as

$$P_\phi \psi = \phi\,(\phi, \psi), \tag{1}$$

   where ϕ and ψ are any two vectors. Show that P_ϕ is a Hermitian operator.

2. (**20 points.**) The requirement for an operator to be unitary is

$$(A\psi, A\psi) = (\psi, \psi) \tag{2}$$

   for any ψ. Show that this implies

$$(A\phi, A\psi) = (\phi, \psi). \tag{3}$$

3. (**20 points.**) Two matrices satisfy the relation

$$AB - BA = 1. \tag{4}$$

   Prove that this cannot be true in a finite dimensional vector space. (Hint: Take trace.)

4. (**20 points.**) Consider the matrix

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{5}$$

   (a) Find all the eigenvalues of the matrix A.

   (b) Find the normalized eigenvectors associated with all the eigenvalues of matrix A.

   (c) Determine the matrix that diagonalizes the matrix A.

5. (**20 points.**) A 3×3 matrix A satisfies the equation

$$A^3 = 1. \tag{6}$$

   Given that the eigenvalues of A are non-degenerate, find all eigenvalues of A.