Midterm Exam No. 02 (Spring 2014)

PHYS 530A: Quantum Mechanics II

Date: 2014 Mar 27

1. (**20 points.**) Using the properties of Pauli matrices,

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k, \tag{1}$$

and the Euler formula

$$e^{ix} = \cos x + i \sin x, \tag{2}$$

evaluate

$$e^{-i\theta \frac{\sigma_x}{2}} \sigma_y e^{i\theta \frac{\sigma_x}{2}}. \tag{3}$$

What is the physical interpretation of this operation?

2. (**20 points.**) The probabilities in a series setup of Stern-Gerlach experiment, for spin-$\frac{1}{2}$, is given by

$$p([A = a'] \to [B = b'] \to [C = c']) = \text{tr}\left(\frac{1 + a'A}{2}\right)\left(\frac{1 + b'B}{2}\right)\left(\frac{1 + c'C}{2}\right), \tag{4}$$

where $[A = a']$ denotes the selection of the beam corresponding to the eigenvalue a'.

   (a) Find the following probabilities:

      i. $p([\sigma_x = +1] \to [\sigma_x = +1])$

      ii. $p([\sigma_x = +1] \to [\sigma_y = +1] \to [\sigma_x = +1])$

   (b) Does the measurement of σ_y *completely* wipe out the prior knowledge of the measurement of σ_x? If yes, why? If no, why not?

   (c) Are σ_x and σ_y complementary?

3. (**20 points.**) An experiment is capable of measuring the following six physical variables:

$$J_x, J_y, J_z, J_x^2, J_y^2, J_z^2, \tag{5}$$

where

$$\mathbf{J} = \frac{\hbar}{2}\boldsymbol{\sigma}. \tag{6}$$

   (a) Out of 15 distinct pairs of variables above, list the pairs that can be measured simultaneously, that is, the measurement of one variable in a pair does not disturb the measurement of the other–the measurements are compatible.

(b) Are the remaining pairs in the list complementary sets? Remember, complementary variables have optimal incompatibility. Complementary variables are pairs that are needed to describe the system, but the measurement of one variable wipes out the prior knowledge of the other.

4. (**40 points.**) Consider the eigenvalue equation

$$\sigma_x|\sigma'_x\rangle = \sigma'_x|\sigma'_x\rangle, \tag{7}$$

where primes denote eigenvalues.

(a) Find the eigenvalues and normalized eigenvectors (up to a phase) of

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{8}$$

For reference we shall call these eigenvectors $|\sigma'_x = +\rangle$ and $|\sigma'_x = -\rangle$.

(b) Now compute the new matrix

$$\bar{\sigma}_x = \begin{pmatrix} \langle\sigma'_x = +|\sigma_x|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_x|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_x|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_x|\sigma'_x = -\rangle \end{pmatrix}. \tag{9}$$

(c) Similarly, compute the new matrices

$$\bar{\sigma}_y = \begin{pmatrix} \langle\sigma'_x = +|\sigma_y|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_y|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_y|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_y|\sigma'_x = -\rangle \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{10}$$

and

$$\bar{\sigma}_z = \begin{pmatrix} \langle\sigma'_x = +|\sigma_z|\sigma'_x = +\rangle & \langle\sigma'_x = +|\sigma_z|\sigma'_x = -\rangle \\ \langle\sigma'_x = -|\sigma_z|\sigma'_x = +\rangle & \langle\sigma'_x = -|\sigma_z|\sigma'_x = -\rangle \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{11}$$

(d) Find the product of the last two matrices, $\bar{\sigma}_y\bar{\sigma}_z$, and express it in terms of $\bar{\sigma}_x$.