Homework No. 09 (Spring 2014)

PHYS 530A: Quantum Mechanics II

Due date: Not applicable

1. We constructed the total angular momentum states of two spin-$\frac{1}{2}$ systems, $j_1 = \frac{1}{2}$, $j_2 = \frac{1}{2}$, by beginning with the total angular momentum state

$$|1, 1\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle_{①} \left|\frac{1}{2}, \frac{1}{2}\right\rangle_{②} \tag{1}$$

and using the lowering operator to construct the $|1, 0\rangle$ and $|1, -1\rangle$ states. The state $|0, 0\rangle$ was then constructed (to within a phase factor) as the state orthogonal to $|1, 0\rangle$.

   (a) Repeat this exercise by beginning with the total angular momentum state $|1, -1\rangle$ and using the raising operator to construct $|1, 0\rangle$ and $|1, 1\rangle$ states.

   (b) Investigate the property of the total angular momentum states under the interchange ①↔②. In particular, find out if each of the total angular momentum states are symmetrical (do not change sign) or antisymmetrical (change sign).

2. Let us construct the total angular momentum states for the composite system built out of two angular momenta $j_1 = 2, j_2 = \frac{1}{2}$.

   (a) Determine the total number of states by counting the individual states,

$$\left(\sum_{m_1=-j_1}^{j_1}\right)\left(\sum_{m_2=-j_2}^{j_2}\right). \tag{2}$$

   Repeat this by counting the number of total angular momentum states,

$$\sum_{j=|j_1-j_2|}^{j_1+j_2} \sum_{m=-j}^{j}. \tag{3}$$

   (b) Beginning with $|5/2, 5/2\rangle$ use the lowering operator to build five other states with $j = 5/2$.

   (c) Construct $|3/2, 3/2\rangle$ state by requiring it to be orthogonal to $|5/2, 3/2\rangle$, and be normalized.

   (d) Beginning with $|3/2, 3/2\rangle$ use the lowering operator to build three other states with $j = 3/2$.

Particle	$\vert T, T_3\rangle$	Q
proton	$\vert\frac{1}{2}, \frac{1}{2}\rangle$	1
neutron	$\vert\frac{1}{2}, -\frac{1}{2}\rangle$	0
$\boldsymbol{\pi}^+$	$\vert 1, 1\rangle$	1
$\boldsymbol{\pi}^0$	$\vert 1, 0\rangle$	0
$\boldsymbol{\pi}^-$	$\vert 1, -1\rangle$	-1

Table 1: Isospin assignments for particles.

3. Construct the total angular momentum state $|1, 1\rangle$ for the composite system built out of two angular momenta $j_1 = 2, j_2 = 1$. Investigate the property of this state under the interchange ①↔②.

4. (Schwinger's QM book, Prob. 3-4a.) Iso(topic) spin T: The nucleon is a particle of isospin $T = \frac{1}{2}$; the state with $T_3 = \frac{1}{2}$ is the proton (p), the state with $T_3 = -\frac{1}{2}$ is the neutron (n). Electric charge of a nucleon is given by $Q = \frac{1}{2} + T_3$. The $\boldsymbol{\pi}$ meson, or pion, has isospin $T = 1$, and electric charge $Q = T_3$, so there are three kinds of pions with different electric charge: $T_3 = 1$ ($\boldsymbol{\pi}^+$), $T_3 = 0$ ($\boldsymbol{\pi}^0$), $T_3 = -1$ ($\boldsymbol{\pi}^-$). (Refer Table 1.)

   Consider the system of a nucleon and a pion. The electric charge of this system is $Q = \frac{1}{2} + T_3$. Check that a system of charge 2, $T_3 = \frac{3}{2}$, is p+$\boldsymbol{\pi}^+$, according to the isospin assignments. Now, if the system is in the state $T = \frac{3}{2}$, $T_3 = \frac{1}{2}$, what is the probability of finding a proton? What is the accompanying $\boldsymbol{\pi}$-meson?