Solutions

Midterm Exam No. 01 (Fall 2014)

PHYS 320: Electricity and Magnetism I

Date: 2014 Sep 12

1. (**20 points.**) (Based on Schwinger et al., problem 7, chapter 1.)
A charge q moves in the vacuum under the influence of uniform fields $\mathbf{E}$ and $\mathbf{B}$. The force on this charge is given by the Lorentz force

$$\mathbf{F} = q\big[\mathbf{E} + \mathbf{v} \times \mathbf{B}\big]. \tag{1}$$

Assume that $\mathbf{E} \cdot \mathbf{B} = 0$ and $\mathbf{v} \cdot \mathbf{B} = 0$.

   (a) At what velocity does the charge move without acceleration, that is, $\mathbf{F} = 0$?

   (b) What is the speed when $\sqrt{\varepsilon_0}|\mathbf{E}| = |\mathbf{B}|/\sqrt{\mu_0}$?
   (Remember, speed of light c in Maxwell's equations is identified using $\varepsilon_0 \mu_0 = 1/c^2$.)

   (c) Give a realization of the physical situation in item (1b) and comment on it intuitively. (This part of the question might not be graded.)

2. (**20 points.**) Show that

$$\boldsymbol{\nabla}(\hat{\mathbf{r}} \cdot \mathbf{a}) = -\frac{1}{r}\,\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}) \tag{2}$$

for a uniform (homogeneous in space) vector $\mathbf{a}$.

3. (**20 points.**) Evaluate the integral

$$\int_{-1}^{1} dx\, \delta(1 - 2x) \Big[8x^2 + 2x - 1\Big]. \tag{3}$$

(Caution: Be careful to avoid a possible error in sign.)

4. (**20 points.**) Evaluate the flux,

$$\int_S d\mathbf{a} \cdot \mathbf{E}, \tag{4}$$

of the uniform (homogeneous in space) field

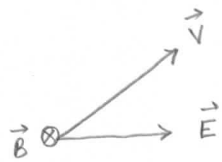
$$\mathbf{E} = E\,\hat{\mathbf{z}} \tag{5}$$

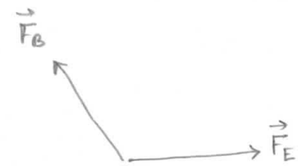
through the surface of a hemispherical bowl of radius R placed on the x-y plane.

5. (**20 points.**) The Heaviside step function, named after Oliver Heaviside (1850-1925), has the integral representation

$$\theta(x) = \int_{-\infty}^{x} dx'\, \delta(x'). \tag{6}$$

MT-01, prob 1





(a) $\vec{F} = 0$

$$\Rightarrow \quad q\vec{E} = -q\,\vec{V}\times\vec{B}$$

$$\Rightarrow \quad \vec{E} = -\vec{V}\times\vec{B}$$

$$\vec{E}\times\vec{B} = -(\vec{V}\times\vec{B})\times\vec{B}$$

$$= \vec{V}\,B^2 - \underbrace{(\vec{V}\cdot\vec{B})}_{=0}\,\vec{B}$$

$$\Rightarrow \quad \vec{V} = \frac{\vec{E}\times\vec{B}}{B^2}$$

$$|\vec{V}| = \frac{E}{B}$$

(b) $|\vec{V}| = \dfrac{E}{B} = \dfrac{1}{\sqrt{\mu_0\epsilon_0}} = c$ = speed of light!

(c) Plane wave, light.

MT-01, prob 2

$$\vec{\nabla}(\hat{r}\cdot\vec{a}) = \vec{\nabla}\left(\frac{1}{r}\,\vec{r}\cdot\vec{a}\right)$$

$$= \underbrace{\left(\vec{\nabla}\frac{1}{r}\right)}_{-\frac{1}{r^2}\vec{\nabla}r = -\frac{\hat{r}}{r^2}}\vec{r}\cdot\vec{a} + \frac{1}{r}\underbrace{(\vec{\nabla}\vec{r})}_{=\overleftrightarrow{1}}\cdot\vec{a} + \frac{1}{r}\underbrace{(\vec{\nabla}\vec{a})}_{=0}\cdot\vec{r}$$

$$= -\frac{\hat{r}}{r^2}\vec{r}\cdot\vec{a} + \frac{1}{r}\overleftrightarrow{1}\cdot\vec{a}$$

$$\overleftrightarrow{1}\cdot\vec{a} = \vec{a}$$

$$= -\frac{1}{r}\left[\hat{r}(\hat{r}\cdot\vec{a}) - \vec{a}\right]$$

$$= -\frac{1}{r}\hat{r}\times(\hat{r}\times\vec{a})$$

MT-01, prob 3

$$\int_{-1}^{1} dx\, \delta(1-2x)\left[8x^2+2x-1\right] = \frac{1}{|-2|}\left[8\left(+\tfrac{1}{2}\right)^2 + 2\left(+\tfrac{1}{2}\right) - 1\right]$$

$$= \frac{1}{2}\left[2+1-1\right]$$

$$= 1$$

MT-01, prob 4

$$d\vec{a} = R^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

$$\hat{r}\cdot\hat{z} = \cos\theta$$

$$\vec{E} = E\hat{z}$$

$$\int_S d\vec{a}\cdot\vec{E} = \int_0^{\frac{\pi}{2}} \sin\theta\, d\theta \int_0^{2\pi} d\phi\; E R^2\, \hat{r}\cdot\hat{z}$$

$$\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta \cos\theta = -\int_1^0 dt\, t = \frac{1}{2}$$

$$= E R^2 \underbrace{\int_0^{2\pi} d\phi}_{2\pi}\; \underbrace{\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta\, \cos\theta}_{\frac{1}{2}}$$

$$= E\pi R^2$$

MT-01, prob 5

$$\theta(x) = \int_{-\infty}^{x} dx' \; \delta(x')$$

(a) $\theta(x) = 0$

(b) $\theta(x) = 1$

(c)
$$\theta(0) = \frac{1}{2}\left[\underset{\epsilon \to 0}{\mathrm{Lt}} \; \theta(x-\epsilon) + \underset{x \to 0}{\mathrm{Lt}} \; \theta(x+\epsilon) \right]$$

$$= \frac{1}{2}\left[0 + 1 \right]$$

$$= \frac{1}{2}$$

(d)