Midterm Exam No. 01 (Fall 2014)

PHYS 320: Electricity and Magnetism I

Date: 2014 Sep 12

1. (**20 points.**) (Based on Schwinger et al., problem 7, chapter 1.)
A charge q moves in the vacuum under the influence of uniform fields $\mathbf{E}$ and $\mathbf{B}$. The force on this charge is given by the Lorentz force

$$\mathbf{F} = q\big[\mathbf{E} + \mathbf{v} \times \mathbf{B}\big]. \tag{1}$$

Assume that $\mathbf{E} \cdot \mathbf{B} = 0$ and $\mathbf{v} \cdot \mathbf{B} = 0$.

   (a) At what velocity does the charge move without acceleration, that is, $\mathbf{F} = 0$?

   (b) What is the speed when $\sqrt{\varepsilon_0}|\mathbf{E}| = |\mathbf{B}|/\sqrt{\mu_0}$?
   (Remember, speed of light c in Maxwell's equations is identified using $\varepsilon_0\mu_0 = 1/c^2$.)

   (c) Give a realization of the physical situation in item (1b) and comment on it intuitively.
   (This part of the question might not be graded.)

2. (**20 points.**) Show that

$$\boldsymbol{\nabla}(\hat{\mathbf{r}} \cdot \mathbf{a}) = -\frac{1}{r}\,\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}) \tag{2}$$

for a uniform (homogeneous in space) vector $\mathbf{a}$.

3. (**20 points.**) Evaluate the integral

$$\int_{-1}^{1} dx\, \delta(1-2x)\Big[8x^2 + 2x - 1\Big]. \tag{3}$$

(Caution: Be careful to avoid a possible error in sign.)

4. (**20 points.**) Evaluate the flux,

$$\int_S d\mathbf{a} \cdot \mathbf{E}, \tag{4}$$

of the uniform (homogeneous in space) field

$$\mathbf{E} = E\,\hat{\mathbf{z}} \tag{5}$$

through the surface of a hemispherical bowl of radius R placed on the x-y plane.

5. (**20 points.**) The Heaviside step function, named after Oliver Heaviside (1850-1925), has the integral representation

$$\theta(x) = \int_{-\infty}^{x} dx'\delta(x'). \tag{6}$$

(a) Evaluate $\theta(x)$ for $x < 0$.

(b) Evaluate $\theta(x)$ for $x > 0$.

(c) What about $\theta(0)$? (This was a question asked by one of us in class.) We could postulate that

$$\theta(0) = \frac{1}{2}\left[\lim_{\varepsilon\to 0}\theta(x-\varepsilon) + \lim_{\varepsilon\to 0}\theta(x+\varepsilon)\right]. \tag{7}$$

Evaluate $\theta(0)$ obtained using Eq. (7).

(d) Plot $\theta(x)$ versus x.