Midterm Exam No. 02 (Fall 2014)

PHYS 320: Electricity and Magnetism I

Date: 2014 Oct 15

1. (**20 points.**) Electric field lines due to four positive charges of equal magnitude placed at the vertices of a square are drawn in Fig. 1. Using Fig. 1 as a guide, estimate the approximate coordinates (choosing the red dot as origin) of *all* the points where a test charge will not experience a force. Also, comment on the stability (or instability) of a test charge kept at these points.

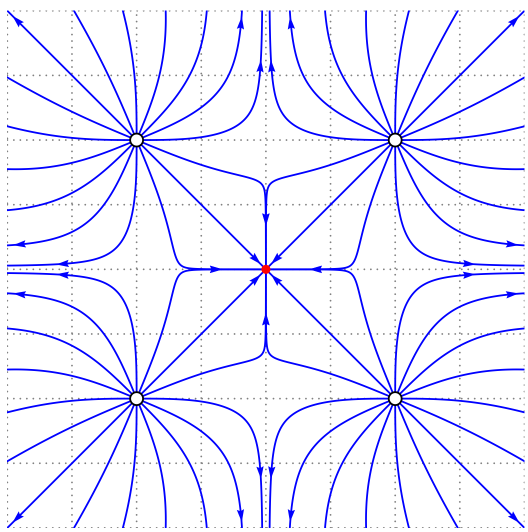

Figure 1: Problem 1.

2. (**20 points.**) The electric field of a point dipole $\mathbf{p}$ is given by the expression

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\Big[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}\Big] = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\mathbf{p}\cdot\mathbf{r})\mathbf{r} - \mathbf{p}r^2}{r^5}\right]. \tag{1}$$

Evaluate $\boldsymbol{\nabla}\cdot\mathbf{E}$.
Hint: Intuitively, the divergence of a vector field is a measure of the density of source/sink of the field. For reference, the electric field lines drawn in Fig. 2 will be those of a point dipole in the limit of distance between the two charges going to zero, keeping the magnitude of the dipole moment fixed.

3. (**20 points.**) Earnshaw's theorem states that Poisson equation does not allow stable configurations for electric field going to zero at infinity. After the lecture on Earnshaw's

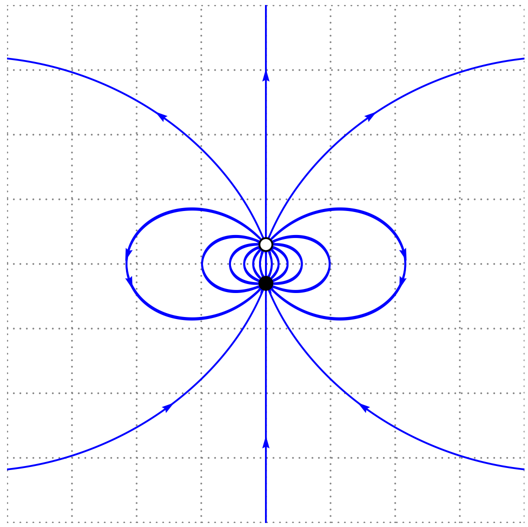

Figure 2: Problem 2.

theorem, in this class, I was part of a conversation that argued the following. What about a test charge placed exactly midway between two positive charges on the x-axis? I answered that the test charge will tend to slip away along the y-axis. Now, what about a test charge placed at the center of six charges, two on each of the axis. I answered that the test charge will tend to slip away in between the axes. Next, what about a test charge placed at the center of a uniformly charged spherical shell. Isn't the charge in a stable configuration now? How would you defend Earnshaw's theorem in this case?
Hint: Remind yourself of the strength of electric field inside a uniformly charged spherical shell.

4. (**20 points.**) Consider a solid sphere of radius R with total charge Q distributed inside the sphere with a charge density

$$\rho(\mathbf{r}) = br^3\,\theta(R-r), \tag{2}$$

where r is the distance from the center of sphere, and $\theta(x) = 1$, if $x > 0$, and 0 otherwise.

   (a) Integrating the charge density over all space gives you the total charge Q. Thus, determine the constant b in terms of Q and R.

   (b) Using Gauss's law find the electric field inside and outside the sphere.

   (c) Plot the electric field as a function of r.

5. (**20 points.**) In class we evaluated the electric potential due to a solid sphere with uniform charge density Q. The angular integral in this evaluation involved the integral

$$\frac{1}{2}\int_{-1}^{1} dt \frac{1}{\sqrt{r^2 + r'^2 - 2rr't}}. \tag{3}$$

Evaluate the integral for $r < r'$ and $r' < r$, where r and r' are distances measured from the center of the sphere. (Hint: Substitute $r^2 + r'^2 - 2rr't = y$.)

6. (**20 points.**) Consider an infinitely thin flat sheet, of infinite extent, constructed out of a continuous distribution of point dipoles, each of individual charge density

$$-\mathbf{p} \cdot \boldsymbol{\nabla} \delta^{(3)}(\mathbf{r} - \mathbf{r}_p), \tag{4}$$

where $\mathbf{r}_p$ is the position of an individual point dipole. The charge density of such a sheet is given by

$$\rho(\mathbf{r}) = -\sigma \frac{\partial}{\partial z} \delta(z), \tag{5}$$

where σ is the electric dipole moment per unit area.

(a) Evaluate the electric potential for the sheet using

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}. \tag{6}$$

(Hint: Use the δ-function property to evaluate the z'-integral, after integrating by parts. The x' and y' integrals can be completed using standard substitutions.)

(b) Evaluate the electric field for the sheet by finding the gradient of the electric potential you calculated using Eq. (6),

$$\mathbf{E}(\mathbf{r}) = -\boldsymbol{\nabla}\phi(\mathbf{r}). \tag{7}$$