Homework No. 01 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Wednesday, 2014 Aug 27, 4:00 PM

1. Problem 1.2, Griffiths 4th edition, ditto in 3rd edition.
2. Find the angle between the face diagonal (1,0,1) and the body diagonal (1,1,1) of the cube in Figure 1.10 of Griffiths 4th edition, ditto in 3rd edition. (Modified version of Example 1.2, Griffiths 4th edition, ditto in 3rd edition.)
3. Problem 1.4, Griffiths 4th edition, ditto in 3rd edition.
4. Using antisymmetric property of Levi-Civita symbol show that

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = -\mathbf{A}\cdot\mathbf{C}\times\mathbf{B}. \tag{1}$$

5. In spherical polar coordinates a point is coordinated by the intersection of family of spheres, cones, and half-planes, given by

$$r = \sqrt{x^2+y^2+z^2}, \tag{2a}$$

$$\theta = \tan^{-1}\sqrt{\frac{x^2+y^2}{z^2}}, \tag{2b}$$

$$\phi = \tan^{-1}\frac{y}{x}, \tag{2c}$$

respectively. Show that the gradient of these surfaces are given by

$$\boldsymbol{\nabla} r = \hat{\mathbf{r}}, \qquad \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{i}} + \sin\theta\sin\phi\,\hat{\mathbf{j}} + \cos\theta\,\hat{\mathbf{k}}, \tag{3a}$$

$$\boldsymbol{\nabla}\theta = \hat{\boldsymbol{\theta}}\,\frac{1}{r}, \qquad \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{i}} + \cos\theta\sin\phi\,\hat{\mathbf{j}} - \sin\theta\,\hat{\mathbf{k}}, \tag{3b}$$

$$\boldsymbol{\nabla}\phi = \hat{\boldsymbol{\phi}}\,\frac{1}{r\sin\theta}, \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}, \tag{3c}$$

which are normal to the respective surfaces. Sketch the surfaces and the corresponding normal vectors. This illustrates that $\boldsymbol{\nabla}$(surface) is a vector (field) normal to the surface.

6. Show that
   (a) $\boldsymbol{\nabla} r = \dfrac{\mathbf{r}}{r} = \hat{\mathbf{r}}$
   (b) $\boldsymbol{\nabla}\,\mathbf{r} = \mathbf{1}$
   (c) $\boldsymbol{\nabla}\cdot\mathbf{r} = 3$

(d) $\boldsymbol{\nabla} \times \mathbf{r} = 0$

7. Show that

   (a) $\boldsymbol{\nabla} \dfrac{1}{r^n} = -\mathbf{r}\, \dfrac{n}{r^{n+2}}$

   (b) $\boldsymbol{\nabla}\, \dfrac{\mathbf{r}}{r^n} = \mathbf{1}\, \dfrac{1}{r^n} - \mathbf{r}\,\mathbf{r}\, \dfrac{n}{r^{n+2}}$

   (c) $\boldsymbol{\nabla} \cdot \dfrac{\mathbf{r}}{r^n} = \dfrac{(3-n)}{r^n}$

   (d) $\boldsymbol{\nabla} \times \dfrac{\mathbf{r}}{r^n} = 0$

   (e) Comment on Problem 1.16, Griffiths 4th edition.

8. Show that

   (a) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

   (b) $\boldsymbol{\nabla} \cdot (\mathbf{A} \times \mathbf{B}) = (\boldsymbol{\nabla} \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\boldsymbol{\nabla} \times \mathbf{B})$