Homework No. 02 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Monday, 2014 Sep 8, 4:00 PM

1. (**10 points.**) A gyroid, see Fig. 1, is an (infinitely connected triply periodic minimal) surface discovered by Alan Schoen in 1970. Schoen presently resides in Carbondale and was a professor at SIU in the later part of his career. Apparently, a gyroid is approximately described by the surface

$$f(x, y, z) = \cos x \sin y + \cos y \sin z + \cos z \sin x \tag{1}$$

when $f(x, y, z) = 0$. Using the fact that the gradient operator

$$\boldsymbol{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z} \tag{2}$$

determines the normal vectors on a surface, evaluate

$$\boldsymbol{\nabla} f(x, y, z). \tag{3}$$

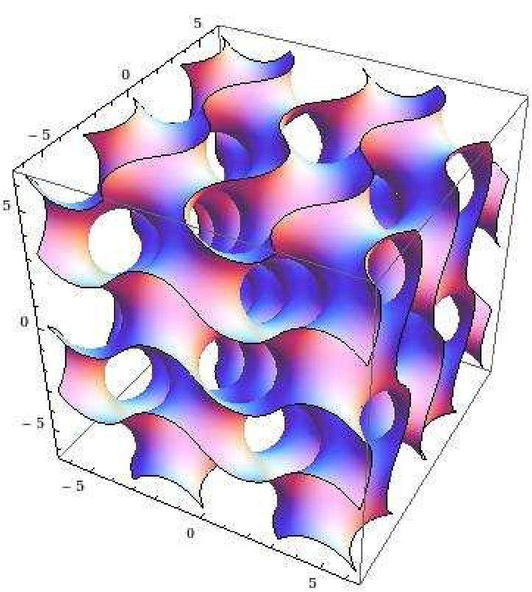

Figure 1: Problem 1.

2. (**30 points.**) (Based on problem 1.26 Griffiths 4th edition.)
Calculate the Laplacian of the following functions:

   (a) $T_a = x^2 + 2xy + 3z + 4$

   (b) $T_b = \sin x \sin y \sin z$

   (c) $\mathbf{v} = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$

3. (**10 points.**) (Based on problem 1.32/1.31 Griffiths 4th/3rd edition.)
Check the fundamental theorem for gradients,

$$\int_{\mathbf{a}}^{\mathbf{b}} d\mathbf{l}\cdot\boldsymbol{\nabla}T = T(\mathbf{b}) - T(\mathbf{a}), \tag{4}$$

using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0,0,0)$, $\mathbf{b} = (1,1,1)$, and the three paths of Fig. 1.28 in Griffiths.

4. (**10 points.**) (Based on problem 1.33/1.32 Griffiths 4th/3rd edition.)
Check the fundamental theorem of divergence,

$$\int_V d^3x\,\boldsymbol{\nabla}\cdot\mathbf{E} = \oint_S d\mathbf{a}\cdot\mathbf{E}, \tag{5}$$

for the vector field $\mathbf{E} = x\,\hat{\mathbf{x}}$. Take a cube of length L as your volume, which is placed with one edge parallel to the x-axis. Using the fact that the divergence of a vector field at a point tells us whether a point is a source or sink of the field, estimate the distribution of the source and sink for the field $\mathbf{E}$?

5. (**10 points.**) (Based on problem 1.34/1.33 Griffiths 4th/3rd edition.)
Check the fundamental theorem of curl,

$$\int_S d\mathbf{a}\cdot\boldsymbol{\nabla}\times\mathbf{E} = \oint_C d\mathbf{l}\cdot\mathbf{E}, \tag{6}$$

(where the sense of the line integration is given by the right hand rule: the contour C is traversed in the sense of the fingers of the right hand and the thumb points in the sense of the orientation of the surface,) for the vector field $\mathbf{E} = y\,\hat{\mathbf{x}} + z\,\hat{\mathbf{y}} + x\,\hat{\mathbf{z}}$. Take a square of length L on the $z = 0$ plane as your surface, which is placed with one side parallel to the x-axis. Using the fact that the curl of a vector field at a point is a measure of the torque experienced by a (point) dipole at the point, estimate the torque field.

6. (**70 points.**) (Based on problem 1.44,45/1.43,44 Griffiths 4th/3rd edition.)
Evaluate the following integrals:

   (a) $\int_2^6 dx\,[3x^2 - 2x - 3]\,\delta(x-3)$

   (b) $\int_{-7}^{7} dx\,\sin x\,\delta(x-\pi)$

   (c) $\int_0^3 dx\,x^3\,\delta(x+1)$

(d) $\int_{-2}^{2} dx\, [3x+3]\, \delta(3x)$

(e) $\int_{-2}^{2} dx\, [3x+3]\, \delta(-3x)$

(f) $\int_{0}^{2} dx\, [3x+3]\, \delta(1-x)$

(g) $\int_{-1}^{1} dx\, 9x^3\, \delta(3x+1)$

7. (**30 points.**) (Based on problem 1.47/1.46 Griffiths 4th/3rd edition.)

(a) Express the charge density $\rho(\mathbf{r})$ of a point charge Q positioned at $\mathbf{r}_a$ in terms of δ-functions. Verify that the volume integral of ρ equals Q.

(b) Express the charge density of an infinitely long wire, of uniform charge per unit length λ and parallel to z-axis, in terms of δ-functions.

(c) Express the charge density of an infinite plate, of uniform charge per unit area σ and parallel to xy-plane, in terms of δ-functions.