Solutions

Homework No. 03 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Monday, 2014 Sep 15, 4:00 PM

1. (**50 points.**) The Maxwell equations, in SI units, are

$$\nabla \cdot \mathbf{D} = \rho, \tag{1}$$
$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$
$$-\nabla \times \mathbf{E} - \frac{\partial}{\partial t}\mathbf{B} = 0, \tag{3}$$
$$\nabla \times \mathbf{H} - \frac{\partial}{\partial t}\mathbf{D} = \mathbf{J}, \tag{4}$$

where

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \tag{5}$$
$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}. \tag{6}$$

The Lorentz force, in SI units, is

$$\mathbf{F} = q\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]. \tag{7}$$

(a) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Gaussian units.

(b) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Lorentz-Heaviside units.

2. (**20 points.**) In Gaussian units the power radiated by an accelerated charged particle of charge e is given by the Larmor formula,

$$P = \frac{2\,e^2}{3\,c^3}\,a^2, \tag{8}$$

where a is the acceleration of the charged particle. Write down the Larmor formula in SI units, and in Lorentz-Heaviside units.

3. (**30 points.**) The fine-structure constant, in Gaussian units,

$$\alpha = \frac{e^2}{\hbar c}, \tag{9}$$

is the parameter that characterizes the strength of the electromagnetic interaction.

HW-03, prob 1

(a) $\rightarrow \vec{\nabla}\cdot\vec{D}_{SI} = \rho_{SI}$

$$\vec{\nabla}\cdot\sqrt{\frac{\epsilon_0}{4\pi}}\,\vec{D}_G = \sqrt{4\pi\epsilon_0}\,\rho_G$$

$$\vec{\nabla}\cdot\vec{D}_G = 4\pi\rho_G$$

$\rightarrow \vec{\nabla}\cdot\vec{B}_{SI} = 0$

$$\vec{\nabla}\cdot\sqrt{\frac{\mu_0}{4\pi}}\,\vec{B}_G = 0 \quad\Rightarrow\quad \vec{\nabla}\cdot\vec{B}_G = 0$$

$\rightarrow \vec{\nabla}\times\vec{E}_{SI} = -\frac{\partial}{\partial t}\vec{B}_{SI}$

$$\vec{\nabla}\times\frac{1}{\sqrt{4\pi\epsilon_0}}\vec{E}_G = -\frac{\partial}{\partial t}\sqrt{\frac{\mu_0}{4\pi}}\,\vec{B}_G \quad\Rightarrow\quad \vec{\nabla}\times\vec{E}_G = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_G$$

$\rightarrow \vec{\nabla}\times\vec{B}_{SI} = \mu_0\frac{\partial}{\partial t}\vec{D}_{SI} + \mu_0\vec{J}_{SI}$

$$\vec{\nabla}\times\sqrt{\frac{\mu_0}{4\pi}}\,\vec{B}_G = \mu_0\frac{\partial}{\partial t}\sqrt{\frac{\epsilon_0}{4\pi}}\,\vec{D}_G + \mu_0\sqrt{4\pi\epsilon_0}\,\vec{J}_G \quad\Rightarrow\quad \vec{\nabla}\times\vec{B}_G = \frac{1}{c}\frac{\partial}{\partial t}\vec{D}_G + \frac{4\pi}{c}\rho_G$$

$\rightarrow \vec{F}_{SI} = q_{SI}\left[\vec{E}_{SI} + \vec{v}\times\vec{B}_{SI}\right]$

$$\vec{F}_G = \sqrt{4\pi\epsilon_0}\,q_G\left[\frac{1}{\sqrt{4\pi\epsilon_0}}\vec{E}_G + \vec{v}\times\sqrt{\frac{\mu_0}{4\pi}}\,\vec{B}_G\right] \quad\Rightarrow\quad \vec{F}_G = q_G\left[\vec{E}_G + \frac{\vec{v}}{c}\times\vec{B}_G\right]$$

(b) $\rightarrow \vec{\nabla}\cdot\vec{D}_{SI} = \rho_{SI}$

$$\vec{\nabla}\cdot\sqrt{\epsilon_0}\,\vec{D}_{LH} = \sqrt{\epsilon_0}\,\rho_{LH} \quad\Rightarrow\quad \vec{\nabla}\cdot\vec{D}_{LH} = \rho_{LH}$$

$\rightarrow \vec{\nabla}\cdot\vec{B}_{SI} = 0$

$$\vec{\nabla}\cdot\sqrt{\mu_0}\,\vec{B}_{LH} = 0 \quad\Rightarrow\quad \vec{\nabla}\cdot\vec{B}_{LH} = 0$$

$\rightarrow \vec{\nabla}\times\vec{E}_{SI} = -\frac{\partial\vec{B}_{SI}}{\partial t}$

$$\vec{\nabla}\times\frac{1}{\sqrt{\epsilon_0}}\vec{E}_{LH} = -\frac{\partial}{\partial t}\sqrt{\mu_0}\,B_{LH} \quad\Rightarrow\quad \vec{\nabla}\times\vec{E}_{LH} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_{LH}.$$

$\rightarrow \vec{\nabla}\times\vec{B}_{SI} = \mu_0\frac{\partial}{\partial t}\vec{D}_{SI} + \mu_0\vec{J}_{SI}$

$$\vec{\nabla}\times\sqrt{\mu_0}\,\vec{B}_{LH} = \mu_0\frac{\partial}{\partial t}\sqrt{\epsilon_0}\,\vec{D}_{LH} + \mu_0\sqrt{\epsilon_0}\,\vec{J}_{LH} \quad\Rightarrow\quad \vec{\nabla}\times\vec{B}_{LH} = \frac{1}{c}\frac{\partial}{\partial t}\vec{D}_{LH} + \frac{1}{c}\vec{J}_{LH}$$

$\rightarrow \vec{F}_{SI} = q_{SI}\left[\vec{E}_{SI} + \vec{v}\times\vec{B}_{SI}\right]$

$$= \sqrt{\epsilon_0}\,q_{LH}\left[\frac{1}{\sqrt{\epsilon_0}}\vec{E}_{LH} + \vec{v}\times\sqrt{\mu_0}\,\vec{B}_{LH}\right] \quad\Rightarrow\quad \vec{F}_{SI} = q_{LH}\left[\vec{E}_{LH} + \frac{\vec{v}}{c}\times\vec{B}_{LH}\right]$$

HW-03, prob 2

$$P_G = \frac{2}{3}\frac{e_G^2}{c^3}a^2$$

$$P_{SI} = \frac{2}{3}\left(\frac{1}{\sqrt{4\pi\epsilon_0}}e_{SI}\right)^2 \frac{1}{c^3}a^2$$

$$= \frac{2e_{SI}^2}{12\pi\epsilon_0 c^3}a^2.$$

$$P_{LH} = \frac{2}{3}\left(\frac{1}{\sqrt{4\pi}}e_{LH}\right)^2 \frac{1}{c^3}a^2$$

$$= \frac{2e_{LH}^2}{12\pi c^3}a^2.$$

HW-03, prob 3

(a) $$\alpha_G = \frac{e_G^2}{\hbar c}$$

$$\alpha_{SI} = \left(\frac{1}{\sqrt{4\pi\epsilon_0}}e_{SI}\right)^2\frac{1}{\hbar c} = \frac{e_{SI}^2}{4\pi\epsilon_0\hbar c}$$

$$\alpha_{LH} = \left(\frac{1}{\sqrt{4\pi}}e_{LH}\right)^2\frac{1}{\hbar c} = \frac{e_{LH}^2}{4\pi\hbar c}$$

(b) SI unit

$$\alpha_{SI} = \frac{e_{SI}^2}{4\pi\epsilon_0\hbar c}$$

$$= \frac{10^{-7}\times(2.99\times10^8)^2\times(1.60\times10^{-19})^2}{\frac{1}{2\pi}\times6.63\times10^{-34}\times2.99\times10^8}$$

$$\approx \frac{1}{137.18} = 0.0073$$

$e_{SI} = 1.60\times10^{-19}$ Coulomb

$$\frac{1}{4\pi\epsilon_0} = \frac{\mu_0}{4\pi}c^2 = 10^{-7}(2.99\times10^8)^2\ \frac{Nm^2}{Coulomb^2}.$$

$$\hbar = \frac{1}{2\pi}\times6.63\times10^{-34}\ Js$$

$$c = 2.99\times10^8\ \frac{m}{s}$$

(b) Gaussian units

$$\alpha_G = \frac{e_G^2}{\hbar c}$$

$e_G = 4.8 \times 10^{-10}$ stat Coulomb

$\hbar = \frac{1}{2\pi} \times 6.63 \times 10^{-34} \times 10^7 \; g \frac{cm^2}{s} = 1.055 \times 10^{-27} \; g \frac{cm^2}{s}$

$c = 2.99 \times 10^{10} \frac{cm}{s}$

$$\alpha_G = \frac{(4.8 \times 10^{-10})^2}{(1.055 \times 10^{-27}) \times (2.99 \times 10^{10})}$$

$$\approx \frac{1}{136.91} = 0.0073$$

Heaviside - Lorentz units

presume that Heaviside-Lorentz units uses meter - kg - second, even though HL units uses CGS.

$$\alpha_{LH} = \frac{e_{LH}^2}{4\pi \hbar c}$$

$e_{LH} = \frac{1}{\sqrt{\epsilon_0}} e_{SI} = \frac{1.60 \times 10^{-19}}{\sqrt{8.85 \times 10^{-12}}} = 5.38 \times 10^{-14}$ charge unit

$\hbar = 1.055 \times 10^{-34}$ J s

$c = 3 \times 10^8 \frac{m}{s}$

$$\alpha_{LH} = \frac{e_{LH}^2}{4\pi \hbar c} = \frac{(5.38 \times 10^{-14})^2}{1.055 \times 10^{-34} \times 4\pi \times (2.99 \times 10^8)} \approx \frac{1}{136.95} \approx 0.0073$$

(c) $\alpha = \frac{e^2}{\hbar c}$

$\frac{1}{\alpha} \approx 137.$