Homework No. 03 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Monday, 2014 Sep 15, 4:00 PM

1. (**50 points.**) The Maxwell equations, in SI units, are

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho, \tag{1}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \tag{2}$$

$$-\boldsymbol{\nabla} \times \mathbf{E} - \frac{\partial}{\partial t}\mathbf{B} = 0, \tag{3}$$

$$\boldsymbol{\nabla} \times \mathbf{H} - \frac{\partial}{\partial t}\mathbf{D} = \mathbf{J}, \tag{4}$$

where

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \tag{5}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}. \tag{6}$$

The Lorentz force, in SI units, is

$$\mathbf{F} = q\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]. \tag{7}$$

(a) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Gaussian units.

(b) Starting from the Maxwell equations and Lorentz force in SI units, derive the corresponding equations in Lorentz-Heaviside units.

2. (**20 points.**) In Gaussian units the power radiated by an accelerated charged particle of charge e is given by the Larmor formula,

$$P = \frac{2\,e^2}{3\,c^3}\,a^2, \tag{8}$$

where a is the acceleration of the charged particle. Write down the Larmor formula in SI units, and in Lorentz-Heaviside units.

3. (**30 points.**) The fine-structure constant, in Gaussian units,

$$\alpha = \frac{e^2}{\hbar c}, \tag{9}$$

is the parameter that characterizes the strength of the electromagnetic interaction.

(a) Write down the corresponding expression for fine-structure constant in SI units, and in Lorentz-Heaviside units.

(b) Verify that the fine-structure constant is a dimensionless quantity. Show that the numerical value of the fine-structure constant is independent of the system of units.

(c) Evaluate the numerical value for the reciprocal of the fine-structure constant, α^{-1}. (A periodic table based on quantum electrodynamics breaks down for atomic numbers greater than α^{-1}.)