Homework No. 04 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Wednesday, 2014 Sep 24, 4:00 PM

Solutions

1. (**40 points.**) Consider a uniformly charged solid sphere of radius R with total charge Q.

   (a) Using Gauss's law show that the electric field inside and outside the sphere is given by

   $$\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0} \dfrac{1}{R^2} \dfrac{r}{R} \hat{\mathbf{r}} & r < R, \\ \dfrac{Q}{4\pi\varepsilon_0} \dfrac{1}{r^2} \hat{\mathbf{r}} & r > R, \end{cases} \tag{1}$$

   where $\mathbf{r}$ is the radial vector with respect to the center of sphere.

   (b) Plot the magnitude of the electric field as a function of r.

   (c) Rewrite your results for the case when the solid sphere is a perfect conductor?

   (d) Rewrite your results for the case of a uniformly charged hollow sphere of radius R with total charge Q.

2. (**40 points.**) Consider an infinitely long and uniformly charged solid cylinder of radius R with charge per unit length λ.

   (a) Using Gauss's law show that the electric field inside and outside the cylinder is given by

   $$\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{\lambda}{2\pi\varepsilon_0} \dfrac{1}{R} \dfrac{r}{R} \hat{\mathbf{r}} & r < R, \\ \dfrac{\lambda}{2\pi\varepsilon_0} \dfrac{1}{r} \hat{\mathbf{r}} & r > R, \end{cases} \tag{2}$$

   where $\mathbf{r}$ is now the radial vector transverse to the axis of the cylinder.

   (b) Plot the magnitude of the electric field as a function of r.

   (c) Rewrite your results for the case when the solid cylinder is a perfect conductor?

   (d) Rewrite your results for the case of a uniformly charged hollow cylinder of radius R with charge per unit length λ.

3. (**30 points.**) Consider a uniformly charged solid slab of infinite extent and thickness $2R$ with charge per unit area σ. (Note that even though the charge is spread out in the whole volume of slab we are describing it using charge per unit area σ.)

HW-04, prob 1

(a) $$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$

$$4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

for $R < r$: $Q_{in} = Q$

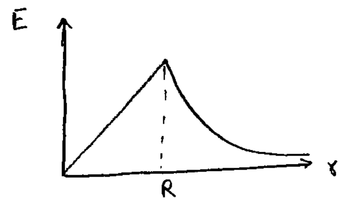
$r < R$: $Q_{in} = Q \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = Q \dfrac{r^3}{R^3}$

Thus,

$$E = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2} Q_{in}$$

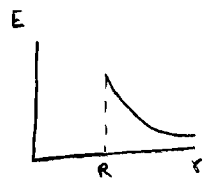
$$= \begin{cases} \dfrac{1}{4\pi\epsilon_0}\dfrac{1}{r^2} Q & \text{if } R < r \\ \dfrac{1}{4\pi\epsilon_0}\dfrac{1}{r^2} Q \dfrac{r^3}{R^3} & \text{if } r < R \end{cases}$$

(b)



(c) $Q_{in} = 0$ for $r < R$ for a conductor. Thus,

$$E = \begin{cases} \dfrac{1}{4\pi\epsilon_0}\dfrac{1}{r^2} Q & \text{if } R < r \\ 0 & \text{if } r < R \end{cases}$$



(d) Same as that for a conductor.

HW-04, prob 2

(a) $$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$

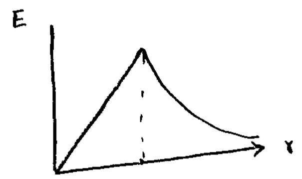
$$2\pi r L \, E = \frac{Q_{in}}{\epsilon_0}$$

for $R < r$: $\quad Q_{in} = \lambda L$

for $r < R$: $\quad Q_{in} = \lambda L \dfrac{\pi r^2}{\pi R^2}$

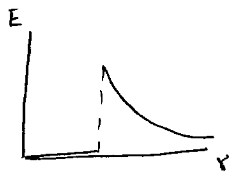
Thus,
$$E = \frac{1}{2\pi r L}\frac{Q_{in}}{\epsilon_0}$$
$$= \begin{cases} \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{1}{r} & \text{for } R < r \\[2ex] \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{1}{r}\dfrac{r^2}{R^2} & \text{for } r < R \end{cases}$$

(b)



(c) $Q_{in} = 0$ for $r<R$ for a conductor. Thus,

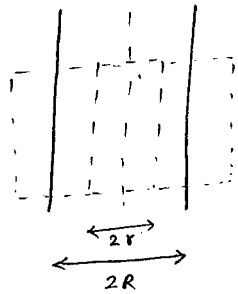
$$E = \begin{cases} \dfrac{\lambda}{2\pi\epsilon_0}\dfrac{1}{r} & \text{for } R < r \\[2ex] 0 & \text{for } r < R \end{cases}$$



(d) Same as (c).

HW-04, prob 3

(a)
$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$



$$\underbrace{EA + EA}_{\text{2 sides.}} = \frac{Q_{in}}{\epsilon_0}$$

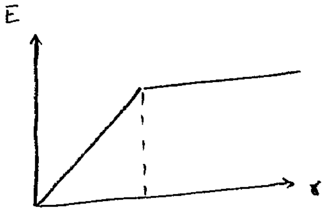
for $R < r$: $Q_{in} = \sigma A$

for $r < R$: $Q_{in} = \sigma A \frac{2r}{2R}$

$$E = \frac{1}{2A} \frac{Q_{in}}{\epsilon_0}$$

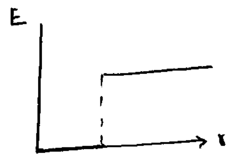
$$= \begin{cases} \frac{\sigma}{2\epsilon_0} & \text{for } R < r \\ \frac{\sigma}{2\epsilon_0} \frac{r}{R} & \text{for } r < R \end{cases}$$

(b)



(c) $Q_{in} = 0$ for $r < R$ for a conductor. Thus,

$$E = \begin{cases} \frac{\sigma}{2\epsilon_0} & \text{for } R < r \\ 0 & \text{for } r < R \end{cases}$$



(d) Same as (c).

HW-04, prob 4

$$Q_{in} = \int d^3r\ \rho(\vec{r})$$

$$= 4\pi \int_0^\infty r^2 dr\ b r\ \theta(R-r)$$

$$= \begin{cases} \cancel{4}\pi b \frac{R^4}{\cancel{4}} & \text{for} \quad R < r \\ \cancel{4}\pi b \frac{r^4}{\cancel{4}} & \text{for} \quad r < R \end{cases}$$

$$\pi b R^4 = Q$$

$$b = \frac{Q}{\pi R^4}$$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$

$$E\ 4\pi r^2 = \frac{Q_{in}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} Q_{in}$$

$$= \begin{cases} \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \pi b R^4 & \text{for} \quad R < r \\ \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \pi b r^4 & \text{for} \quad r < R \end{cases}$$

$$= \begin{cases} \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} & \text{for} \quad R < r \\ \frac{Q}{4\pi\epsilon_0} \frac{r^2}{R^4} & \text{for} \quad r < R \end{cases}$$

HW-04, prob 5

$$Q_{in} = \int_R d^3r\ \rho(\vec{r}) = 4\pi \int_0^R r^2 dr \frac{\sigma}{r} = 4\pi\sigma \frac{R^2}{2}$$

$$\oint_R \vec{E} \cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$

$$\cancel{4\pi} R^2 E = \cancel{4\pi}\sigma \frac{\cancel{R^2}}{2} \frac{1}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

HW-04, prob 6

$$Q_{in} = \int d^3r\, \rho(\vec{r}) = 4\pi \int_0^\infty \cancel{r^2} dr\, \frac{k}{\cancel{r^2}}\, \theta(a-r)\,\theta(r-b)$$

$$= 4\pi k \int_0^\infty dr\, \theta(a-r)\,\theta(r-b)$$

$$= \begin{cases} 0 & \text{for} \quad 0 \leq r < a \\ 4\pi k\,(r-a) & \text{for} \quad a < r < b \\ 4\pi k\,(b-a) & \text{for} \quad b < r < \infty \end{cases}$$

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q_{in}}{\epsilon_0}$$

$$E = \begin{cases} 0 & 0 \leq r < a \\ \frac{k}{\epsilon_0}\frac{1}{r^2}(r-a) & a < r < b \\ \frac{k}{\epsilon_0}\frac{1}{r^2}(b-a) & b < r < \infty \end{cases}$$

Using total charge $4\pi k(b-a) = Q$ we have $k = \frac{Q}{4\pi k(b-a)}$

$$E = \begin{cases} 0 & 0 \leq r < a \\ \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\frac{r-a}{b-a} & a < r < b \\ \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2} & b < r < \infty \end{cases}$$