Homework No. 10 (2014 Fall)

PHYS 320: Electricity and Magnetism I

Due date: Wednesday, 2014 Dec 3, 4:00 PM

1. (**10 points.**) Using the series representation for Bessel functions,

$$J_m(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(m+n)!} \left(\frac{t}{2}\right)^{m+2n}, \tag{1}$$

prove the relation

$$J_m(t) = (-1)^m J_{-m}(t). \tag{2}$$

Hint: Break the sum on n into two parts. Note that the gamma function $\Gamma(z)$, which generalizes the factorial,

$$n! = \Gamma(n+1), \qquad \Gamma(z+1) = z\Gamma(z), \tag{3}$$

beyond positive integers, satisfies

$$\frac{1}{\Gamma(-k)} = 0 \quad \text{for} \quad k = 0, 1, 2, \ldots. \tag{4}$$

2. (**20 points.**) Use the integral representation of $J_m(t)$,

$$i^m J_m(t) = \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{it\cos\alpha - im\alpha}, \tag{5}$$

to prove the recurrence relations

$$2\frac{d}{dt} J_m(t) = J_{m-1}(t) - J_{m+1}(t), \tag{6a}$$

$$2\frac{m}{t} J_m(t) = J_{m-1}(t) + J_{m+1}(t). \tag{6b}$$

3. (**10 points.**) Using the recurrence relations of Eq. (6), show that

$$\left(-\frac{d}{dt} + \frac{m-1}{t}\right)\left(\frac{d}{dt} + \frac{m}{t}\right) J_m(t) = \left(\frac{d}{dt} + \frac{m+1}{t}\right)\left(-\frac{d}{dt} + \frac{m}{t}\right) J_m(t) = J_m(t) \tag{7}$$

and from this derive the differential equation satisfied by $J_m(t)$.