Date: 2014 Sep 8

1. Maxwell's equations (in integral form, in SI units)

(i) Gauss's law

$$\oint d\vec{a} \cdot \vec{E} = \frac{Q_{in}}{\epsilon_0}$$

(ii) Absence of magnetic charge.

$$\oint d\vec{a} \cdot \vec{B} = 0$$

$$\phi_B = \int_S d\vec{a} \cdot \vec{B}$$

$$\phi_E = \int_S d\vec{a} \cdot \vec{E}$$

(iii) Faraday's law

$$-\oint d\vec{l} \cdot \vec{E} = \frac{\partial}{\partial t} \phi_B$$

(iv) Ampere's law

$$\oint d\vec{l} \cdot \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi_E + \mu_0 I_{in}$$

In addition we have the Lorentz force

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

(2) Charge density:

$$Q(t) = \int d^3r \, \rho(\vec{r}, t)$$

$\rho = \dfrac{\text{charge}}{\text{volume}}$

(3) Charge flux density (or current density)

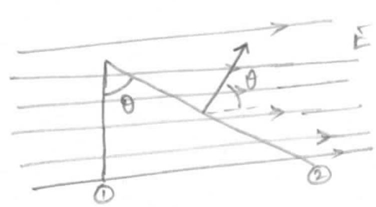
$$\vec{j}(\vec{r}, t) = \vec{v} \, \rho(\vec{r}, t)$$

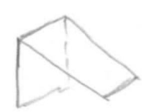
(4) Flux of charge. (or current)

$$I = \int_A d\vec{a} \cdot \vec{j}$$

(5) Flux:

— measure of amount of field lines crossing an area.





$$\phi_E = \int_S d\vec{a} \cdot \vec{E}$$

$$\phi_E^{①} = EA$$

$$\phi_E^{②} = EA \cos\theta$$

→ Area is the magnitude of the vector normal to the area (for infinitely small area).

6 Gauss's law

$$\oint d\vec{a} \cdot \vec{E} = \frac{1}{\epsilon_0} Q_{in}$$

$$\int_V d^3r \; \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \int_V d^3r \; \rho$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho.$$

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{D} = \epsilon_0 \vec{E}$$

2nd & 3rd step uses the fact that the integral is true for an arbitrary area.

7 Absence of magnetic charge.

$$\oint d\vec{a} \cdot \vec{B} = 0$$

$$\int_V d^3r \; \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

8 Faraday's law

$$-\oint d\vec{l} \cdot \vec{E} = \frac{\partial}{\partial t} \phi_B$$

$$-\int_S d\vec{a} \cdot \vec{\nabla} \times \vec{E} = \frac{\partial}{\partial t} \int_S d\vec{a} \cdot \vec{B}$$

$$-\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

(9) Ampere's law

$$\oint d\vec{l} \cdot \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \Phi_E + \mu_0 I_{in}$$

$$\int_S d\vec{a} \cdot \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int_S d\vec{a} \cdot \vec{E} + \mu_0 \int_S d\vec{a} \cdot \vec{J}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

$$\vec{\nabla} \times \left(\frac{1}{\mu_0} \vec{B}\right) = \frac{\partial}{\partial t} \epsilon_0 \vec{E} + \vec{J}$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial}{\partial t} \vec{D} + \vec{J}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

(10) Thus, the Maxwell equations in differential form are.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$-\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \frac{\text{Tesla-m}}{\text{Ampere}}.$$

$$c = 299\ 729\ 458 \ \frac{m}{s}$$

(EXACT)

$\vec{P} \to$ electric polarization

$\vec{M} \to$ magnetic polarization

$$\epsilon_0 \mu_0 = \frac{1}{c^2}$$

(11) Maxwell's equations in Gaussian units

$$\vec{\nabla} \cdot \vec{D} = 4\pi \rho$$

$$-\vec{\nabla} \times \vec{E} = \frac{1}{c}\frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

$$\vec{D} = \vec{E} + 4\pi \vec{P}$$

$$\vec{H} = \vec{B} - 4\pi \vec{M}$$

$$\vec{F} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

(12) Maxwell's equations in Lorentz-Heaviside units

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$-\vec{\nabla} \times \vec{E} = \frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

$$\vec{D} = \vec{E} + \vec{P}$$

$$\vec{H} = \vec{B} - \vec{M}$$

$$\vec{F} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

(13) Conversions

$$\vec{D}_G = \sqrt{\frac{4\pi}{\epsilon_0}}\,\vec{D}_{SI} = \sqrt{4\pi}\,\vec{D}_{LH}$$

$$\vec{E}_G = \sqrt{4\pi\epsilon_0}\,\vec{E}_{SI} = \sqrt{4\pi}\,\vec{E}_{LH}$$

$$\vec{H}_G = \sqrt{4\pi\mu_0}\,\vec{H}_{SI} = \sqrt{4\pi}\,\vec{H}_{LH}$$

$$\vec{B}_G = \sqrt{\frac{4\pi}{\mu_0}}\,\vec{B}_{SI} = \sqrt{4\pi}\,\vec{B}_{LH}$$

$$\rho_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\,\rho_{SI} = \frac{1}{\sqrt{4\pi}}\,\rho_{LH}$$

$$\vec{P}_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\,\vec{P}_{SI} = \frac{1}{\sqrt{4\pi}}\,\vec{P}_{LH}$$

$$\vec{J}_G = \frac{1}{\sqrt{4\pi\epsilon_0}}\,\vec{J}_{SI} = \frac{1}{\sqrt{4\pi}}\,\vec{J}_{LH}$$

$$\vec{M}_G = \sqrt{\frac{\mu_0}{4\pi}}\,\vec{M}_{SI} = \frac{1}{\sqrt{4\pi}}\,\vec{M}_{LH}$$

14 Example

$$\vec{\nabla} \times \vec{B}_{SI} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{E}_{SI} + \mu_0 \vec{J}_{SI}$$

$$\vec{\nabla} \times \sqrt{\frac{\mu_0}{4\pi}}\, \vec{B}_G = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \frac{1}{\sqrt{4\pi \epsilon_0}} \vec{E}_G + \mu_0 \sqrt{4\pi \epsilon_0}\, \vec{J}_G$$

$$\vec{\nabla} \times \vec{B}_G = \sqrt{\mu_0 \epsilon_0} \frac{\partial}{\partial t} \vec{E}_G + 4\pi \sqrt{\mu_0 \epsilon_0}\, \vec{J}_G$$

$$\vec{\nabla} \times \vec{B}_G = \frac{1}{c} \frac{\partial}{\partial t} \vec{E}_G + \frac{4\pi}{c} \vec{J}_G$$