

Date: 2014 Sep 15

① Coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}$$

② Electric field

$$\vec{F} = q \vec{E}$$

③ Electric field due to point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_a}{|\vec{r} - \vec{r}_a|^2} \frac{\vec{r} - \vec{r}_a}{|\vec{r} - \vec{r}_a|}$$

④ Superposition principle

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 + \dots$$

⑤ Example:

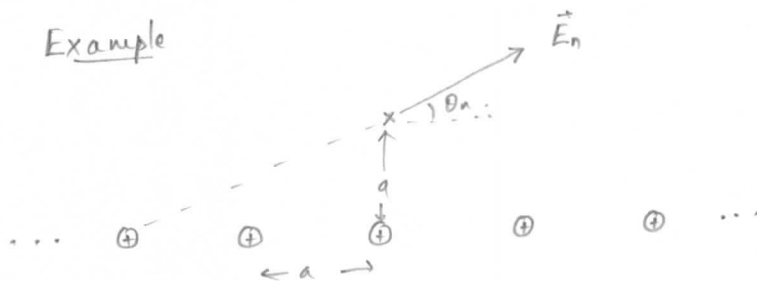


$$\begin{aligned} \vec{E}_{\text{tot}} &= \hat{i} \left[\frac{1}{4\pi\epsilon_0} \frac{q}{a^2} + \frac{1}{4\pi\epsilon_0} \frac{q}{(2a)^2} + \frac{1}{4\pi\epsilon_0} \frac{q}{(3a)^2} + \dots \right] \\ &= \hat{i} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \left[1 + \frac{1}{2^2} + \dots \right] \\ &= \hat{i} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \\ &= \hat{i} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \frac{\pi^2}{6} \end{aligned}$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

Zeta function

⑥ Example



$$\vec{E} = \dots + \vec{E}_{-2} + \vec{E}_{-1} + \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \dots$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2 + n^2 a^2} \sin \theta_n \hat{j}$$

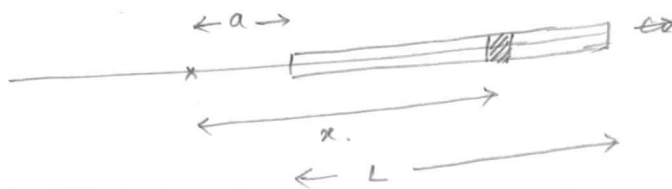
$$= \sum_{n=-\infty}^{+\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{a^2 + n^2 a^2} \frac{a}{\sqrt{a^2 + n^2 a^2}} \hat{j}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \sum_{n=-\infty}^{+\infty} \frac{1}{(1+n^2)^{\frac{3}{2}}} \hat{j}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} C \hat{j}$$

$$C = \sum_{n=-\infty}^{+\infty} \frac{1}{(1+n^2)^{\frac{3}{2}}}$$

⑦ Continuous line charge.



$$\lambda = \frac{Q}{L} = \frac{dq}{dx}$$

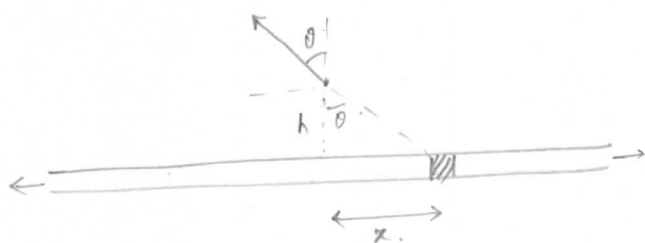
$$d\vec{E} = -\hat{i} \frac{1}{4\pi\epsilon_0} \frac{dq}{x^2}$$

$$\vec{E} = \int d\vec{E} = -\hat{i} \frac{1}{4\pi\epsilon_0} \int_a^{a+L} \frac{\lambda dx}{x^2}$$

$$= -\hat{i} \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \left[\frac{1}{a} - \frac{1}{a+L} \right]$$

$$= -\hat{i} \frac{1}{4\pi\epsilon_0} \frac{Q}{a(a+L)} \xrightarrow{L \ll a} -\hat{i} \frac{1}{4\pi\epsilon_0} \frac{Q}{a^2}$$

⑧

Infinite line charge

$$\lambda = \frac{Q}{L} = \frac{dq}{dx}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{h^2 + x^2}$$

$$d\vec{E} = dE [-\sin\theta \hat{i} + \cos\theta \hat{j}]$$

$$\vec{E} = \hat{j} \int_{x=-\infty}^{+\infty} \frac{dq}{h^2 + x^2} \frac{1}{4\pi\epsilon_0} \frac{h}{\sqrt{h^2 + x^2}}$$

$$= \hat{j} \int_{x=-\infty}^{+\infty} \frac{\lambda dx}{(h^2 + x^2)^{3/2}} \frac{h}{4\pi\epsilon_0}$$

$$= \hat{j} \frac{\lambda h}{4\pi\epsilon_0} \int_{\theta=-\pi/2}^{\pi/2} \frac{h \sec^2\theta d\theta}{h^3 \sec^3\theta}$$

$$= \hat{j} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{h} \underbrace{\int_{-\pi/2}^{\pi/2} \cos\theta d\theta}_{=2}$$

$$= \hat{j} \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{h}$$

$$x = h \tan\theta$$

$$dx = h \sec^2\theta d\theta$$

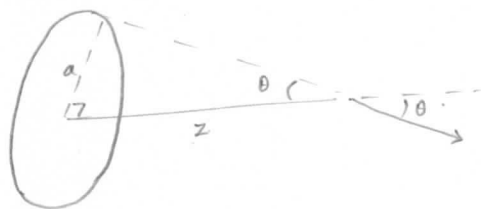
→ Even though there is infinite charge E is finite. Due to $\frac{1}{r^2}$ decay of E .

$$\rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \text{finite}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \infty$$

9

Ring



$$d\vec{E} = dE [\cos\theta \hat{z} + \sin\theta \hat{z}_\perp]$$

$$\begin{aligned} \vec{E} &= \hat{z} \frac{\cos\theta}{4\pi\epsilon_0} \int_0^{2\pi} \frac{dq}{a^2+z^2} \\ &= \hat{z} \frac{1}{4\pi\epsilon_0} \frac{z}{(a^2+z^2)^{\frac{3}{2}}} Q \\ &= \hat{z} \frac{1}{4\pi\epsilon_0} Q \frac{z}{(a^2+z^2)^{\frac{3}{2}}} \end{aligned}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{a^2+z^2}$$

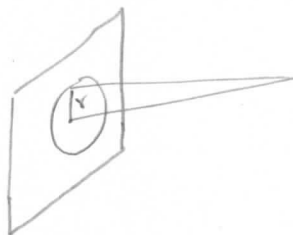
$$dx = a d\phi$$

$$\frac{dq}{dx} = \frac{Q}{2\pi a}$$

$$\begin{aligned} dq &= \frac{Q dx}{2\pi a} \\ &= \frac{Q}{2\pi} d\phi \end{aligned}$$

10

Infinite plate



$$\begin{aligned} \vec{E} &= \hat{z} \frac{1}{4\pi\epsilon_0} \int_{r=0}^{\infty} dq \frac{z}{(r^2+z^2)^{\frac{3}{2}}} \\ &= \hat{z} \frac{1}{4\pi\epsilon_0} z 2\pi \int_{r=0}^{\infty} \frac{r dr}{(r^2+z^2)^{\frac{3}{2}}} \\ &= \hat{z} \frac{1}{4\pi\epsilon_0} z 2\pi \int_z^{\infty} \frac{dt}{t^{\frac{3}{2}}} \\ &= \hat{z} \frac{z}{4\epsilon_0} 2 \left[\frac{1}{z} - 0 \right] = \hat{z} \frac{\sigma}{2\epsilon_0} \end{aligned}$$

$$\frac{dQ}{2\pi r dr} = \sigma = \frac{\text{charge}}{\text{area}}$$

$$\begin{aligned} t &= r^2 + z^2 \\ dt &= 2r dr \end{aligned}$$

→ acceleration due to gravity is constant near Earth

→ point → $\frac{kQ}{r^2}$
line charge → $k \frac{\lambda}{r}$
plate → $k 2\pi \sigma$