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① Vectors

$$\vec{A} = a_1 \hat{e}_1 + a_2 \hat{e}_2 + \dots$$

$$\vec{A}^i = \sum_{m=1}^N a_m \hat{e}_m^{(i)}$$

Or orthogonality:

$$\hat{e}_m \cdot \hat{e}_n = \delta_{mn}$$

$$\sum_{i=1}^N \hat{e}_m^{(i)} \hat{e}_n^{(i)} = \delta_{mn}$$

Completeness

$$\sum_{m=1}^N \hat{e}_m^{(i)} \hat{e}_m^{(j)} = \overset{\leftrightarrow}{1}^{(ij)}$$

② Let us derive the completeness relation above. Let

→ tensor generalization of

$$\vec{A}^{(i)} = \sum_{m=1}^N a_m \hat{e}_m^{(i)}$$

$$\overset{\leftrightarrow}{1}^{(ij)} = \sum_{m=1}^N \vec{a}_m^{(i)} \hat{e}_m^{(j)}$$

$$\overset{\leftrightarrow}{1}^{(ij)} \cdot \hat{e}_n^{(i)} = \sum_{m=1}^N \vec{a}_m^{(i)} \hat{e}_m^{(j)} \hat{e}_n^{(i)}$$

$$\hat{e}_n^{(i)} = \sum_{m=1}^N \vec{a}_m^{(i)} \delta_{mn}$$

$$= \vec{a}_n^{(i)}$$

$$\Rightarrow \overset{\leftrightarrow}{1}^{(ij)} = \sum_{m=1}^N \hat{e}_m^{(i)} \hat{e}_m^{(j)}$$

- ③ Completeness relation implies that any vector can be expressed in terms of some components.

$$\vec{1}^{(ij)} = \sum_{m=1}^N \hat{e}_m^{(i)} \hat{e}_m^{(j)}$$

$$\vec{1}^{(ij)} \cdot \vec{A}^{(i)} = \sum_{m=1}^N \hat{e}_m^{(i)} \underbrace{\hat{e}_m^{(i)} \cdot \vec{A}^{(i)}}_{a_m} \rightarrow \text{direction cosine.}$$

$$\vec{A}^{(i)} = \sum_{m=1}^N a_m \hat{e}_m^{(i)}$$

④ Fourier space (discrete)

→ Eigen vector : $e^{im\phi}$, $0 \leq \phi < 2\pi$, $m = 0, \pm 1, \pm 2, \dots$

Usually the eigenvectors are solutions of a differential equation.

→ Orthogonality :

$$\int_0^{2\pi} \frac{d\phi}{2\pi} e^{im\phi} e^{-in\phi} = \delta_{mn}$$

→ can be verified by explicit integration.

→ Completeness :

$$\text{Let } \delta(\phi - \phi') = \sum_{m=-\infty}^{+\infty} a_m(\phi') e^{im\phi}$$

↓ use orthogonality

$$\delta(\phi - \phi') = \frac{1}{2\pi} \sum_{m=-\infty}^{+\infty} e^{-im\phi'} e^{im\phi}$$

$$\text{to derive } a_m(\phi') = \frac{1}{2\pi} e^{-im\phi'}$$