Date: 2014 Oct 29

Dielectric models and response functions

1. The electric properties of a material has been contained in the polarization vector. We considered cases in which materials had a permanent polarization. But, an external electric field could induce a polarization in the material.

2. The polarization vector is defined as.

$$\vec{P}(\vec{r},t) = n(\vec{r})\, \vec{d}(\vec{r},t) = n(\vec{r})\, e\, \vec{r}$$

$n = \dfrac{\text{atoms}}{\text{volume}}$. $\vec{d} = e\vec{r}$

3. Also, the equation

$$\vec{\nabla}\cdot(\epsilon_0 \vec{E}) = \rho - \vec{\nabla}\cdot\vec{P}$$

suggests the definition

$$\epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\Rightarrow \quad \vec{P} = \epsilon_0 \left(\frac{\epsilon}{\epsilon_0} - 1\right)\vec{E} = \epsilon_0 \chi \vec{E}$$

ϵ – dielectric permittivity

χ – electric susceptibility

which clarifies the idea that the polarization vector is a response to an applied electric field.

(4) In general we could have.

$$\vec{P}(\vec{x},t) = \epsilon_0 \int d^3x' \int_{-\infty}^{+\infty} dt' \, \chi(\vec{x},\vec{x}',t,t') \, \vec{E}(\vec{x}',t')$$

$\downarrow$ only the local electric field contributes. To the leading order, at least.

$$= \epsilon_0 \int_{-\infty}^{+\infty} dt' \, \chi(\vec{x},t,t') \, \vec{E}(\vec{x},t')$$

$\downarrow$ translational invariance in time

$$= \epsilon_0 \int_{-\infty}^{+\infty} dt' \, \chi(\vec{x},t-t') \, \vec{E}(\vec{x},t')$$

$\downarrow$ causal

$$= \epsilon_0 \int_{-\infty}^{\infty} dt' \, \chi(\vec{x},t-t') \, \vec{E}(\vec{x},t')$$

(5) In the frequency domain, which is what is more physical for this case, we have.

$$\vec{P}(\vec{x},\omega) = \epsilon_0 \, \chi(\omega) \, \vec{E}(\vec{x},\omega)$$

6 Properties

(i) $\chi(t)$ should be real.

$$\chi(\omega) = \chi^*(-\omega)$$

(ii)
$$\mathrm{Re}\,\chi(\omega) = \underset{\delta\to 0}{\mathrm{Lt}} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi}\, 2\left[\mathrm{Im}\,\chi(\omega')\right] \frac{(\omega'-\omega)}{(\omega'-\omega)^2+\delta^2}$$

which is the Kramers-Kronig relation.

7 To obtain a reasonable estimate we consider the Fermi model

$$m\frac{d^2}{dt^2}\vec{x}(t) = q\vec{E} - m\omega_0^2\vec{x} - m\gamma\frac{d\vec{x}}{dt}.$$

to bind the charge to the atom

to characterize the dissipation or friction.

In the frequency space we have.

$$\frac{d}{dt} \to -i\omega$$

$$\vec{x}(\omega) = \frac{\frac{q}{m}}{\omega_0^2 - i\gamma\omega - \omega^2}\,\vec{E}(\omega)$$

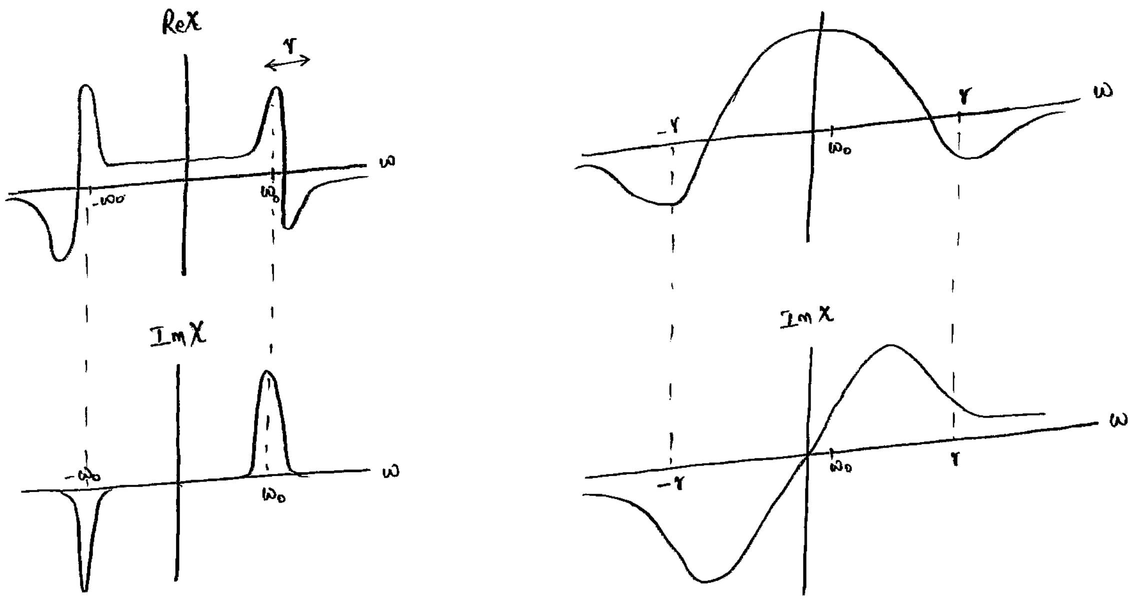
$$\vec{P}(\omega) = nq\vec{x} = \epsilon_0 \frac{\frac{nq^2}{m\epsilon_0}}{\omega_0^2 - i\gamma\omega - \omega^2}\,\vec{E}(\omega)$$

8 Comparison with 3 lets us identify

$$\chi(\omega) = \frac{\omega_p^2}{\omega_0^2 - i\gamma\omega - \omega^2}$$

$$= \underbrace{\frac{\omega_p^2(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}}_{Re\,\chi} + i\underbrace{\frac{\omega_p^2\,\gamma\,\omega}{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}}_{Im\,\chi}.$$

9 Insulator ($\gamma \ll \omega_0$) — Conductor ($\omega_0 \ll \gamma$)



→ Fluctuation - dissipation theorem

→ Johnson noise.

→ Anderson localization