Midterm 01 (PHYS - 205B, Spring 2015)

MT-01, prob. 1

(a)

$$\frac{k q^2}{R^2} = \frac{G m_E m_m}{R^2}$$

$$q = \sqrt{\frac{G m_E m_m}{k}} = \sqrt{\frac{6.7\times10^{-11}\times6\times10^{24}\times7\times10^{22}}{9\times10^{9}}} \sim 6\times10^{13}\,C$$

(b)

$$1\,kg \text{ of electrons} = \frac{1\,kg}{9.1\times10^{-31}\,kg} \text{ number of electrons}$$

$$= 10^{30} \text{ number of electrons}$$

$$= 10^{30}\times1.6\times10^{-19}\,C$$

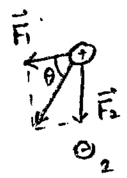
$$= 1.6\times10^{11}\,C$$

Thus,

$$q = 6\times10^{13}\,C \times \frac{1kg \text{ of electrons}}{1.6\times10^{11}\,C}$$

$$= 375\ kg \text{ of electrons.}$$

MT-01, prob. 2



$$\vec{F_1} = -\frac{k q_1 q}{(0.300)^2}\hat{i} = -\frac{9\times10^{9}\times6\times10^{-9}\times4\times10^{-9}}{(0.300)^2}\hat{i}$$

$$= -2.4\times10^{-6}\ \hat{i}\ N$$

$$\vec{F_2} = -\frac{k q_2 q}{(0.100)^2} = -\frac{9\times10^{9}\times3\times10^{-9}\times4\times10^{-9}}{(0.100)^2}\hat{j}$$

$$= -10.8\times10^{-6}\ \hat{j}\ N$$

$$F_{tot} = \sqrt{F_1^2+F_2^2} = 11.1\times10^{-6}\ N$$

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{10.8}{2.4}\right) = 77.5^\circ \text{ w.r.t negative x-axis.}$$

MT-01, prob 3

A	B	C
Q	Q	0
$\frac{Q}{2}$	Q	$\frac{Q}{2}$
$\frac{Q}{2}$	$\frac{Q+\frac{Q}{2}}{2}=\frac{3}{4}Q$	$\frac{Q+\frac{Q}{2}}{2}=\frac{3}{4}Q$

(a) $\frac{Q}{2}$

(b) $\frac{3}{4}Q$

(c) $F = \frac{kQ^2}{R^2}$

$$F_{new} = \frac{k\left(\frac{Q}{2}\right)\left(\frac{3}{4}Q\right)}{R^2} = \frac{3}{8}\frac{kQ^2}{R^2} = \frac{3}{8}F.$$

MT-01, prob 4

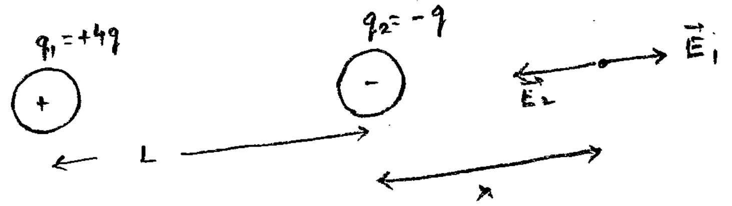
(a)
$$\vec{F} = q\vec{E}$$
$$= 6\times10^{-5}\,C\left[2.30\times10^{3}\,\frac{N}{C}\,\hat{i} + 570\,\frac{N}{C}\,\hat{j} + 0\,\hat{k}\right]$$
$$= 0.138\,N\,\hat{i} + 0.0342\,N\,\hat{j} + 0\,\hat{k}$$

$m = 13\times10^{-3}$ kg

(b)
$$\vec{a} = \frac{\vec{F}}{m} = \underbrace{1.06\times10^{-5}\,\frac{m}{s^2}}_{a_x}\,\hat{i} + \underbrace{0.263\times10^{-5}\,\frac{m}{s^2}}_{a_y}\,\hat{j} + 0\,\hat{k}$$

$$\vec{x} = \frac{1}{2}a_x t^2\,\hat{i} + \frac{1}{2}a_y t^2\,\hat{j} + 0\,\hat{k}$$
$$= 4.77\times10^{-5}\,m\,\hat{i} + 1.18\times10^{-5}\,m\,\hat{j} + 0\,\hat{k}$$

MT-01, prob 5



$$E_1 = E_2$$

$$\frac{k q_1}{(L+x)^2} = \frac{k q_2}{x^2}$$

$$\frac{4}{(L+x)^2} = \frac{1}{x^2} \Rightarrow \pm 2x = L + x$$

$$x = \frac{L}{(-1 \pm 2)} = L, \ -\frac{L}{3}$$ → not consistent with x positive.

$x = L$

cooridinate w.r.t origin $= L + L = 2L$.

MT-01, prob 6

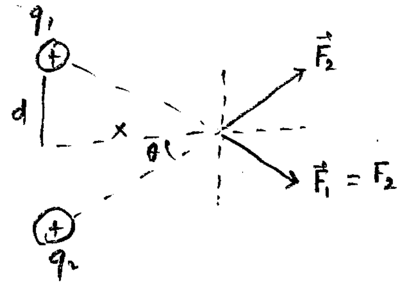
(a) proton moves in the direction of $\vec{E}$ and electron moves in the opposite direction of $\vec{E}$.

(b) $V_e = V_0 + a_e t = 0 + \frac{eE}{m_e} t \Rightarrow \frac{V_e}{V_p} = \frac{m_p}{m_e} = 1836$

$V_p = V_0 + a_p t = 0 + \frac{eE}{m_p} t$

(c) $\frac{P_e}{P_p} = \frac{m_e V_e}{m_p V_p} = 1$ → both have same momentum.

MT-01, prob 7



$$\vec{F}_{tot} = 2 F_1 \cos\theta \; \hat{i}$$

$$= 2k \frac{q_1 q_3}{d^2+x^2} \frac{x}{(d^2+x^2)^{\frac{1}{2}}} \hat{i}.$$

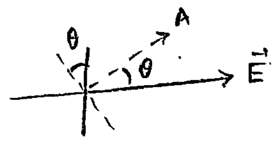
$$F = 2kq_1q_3 \frac{x}{(d^2+x^2)^{\frac{3}{2}}}$$

$$\frac{\partial F}{\partial x} = 2kq_1q_3 \left[\frac{1}{(d^2+x^2)^{\frac{3}{2}}} - \frac{3}{\not{2}} \frac{x \not{2} x}{(d^2+x^2)^{\frac{5}{2}}} \right]$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow \quad d^2 + x^2 - 3x^2 = 0$$

$$x = \frac{d}{\sqrt{2}} = \frac{16.0\,cm}{\sqrt{2}} = 11.3\ cm.$$

MT-01, prob 8



$$\phi_E = EA\cos 0 = 75\ \frac{N}{C} m^2$$

$$\phi_E' = EA\cos 35$$

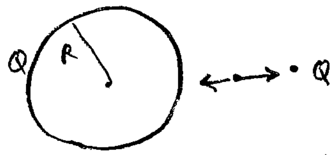
$$= 75 \times \cos 35 = 61.44\ \frac{N}{C} m^2.$$

MT-01, prob 9

$$\phi_E = \frac{Q_{inside}}{\epsilon_0} = \frac{8.85\times 10^{-12}\,C}{8.85\times 10^{-12}\,\frac{C^2}{Nm^2}}$$

$$\approx 1 \;\frac{N}{C}\,m^2$$

MT-01, prob 10



$$\vec{E}_{tot} = \hat{i}\,\frac{kQ}{\left(\frac{3}{2}R\right)^2} - \hat{i}\,\frac{kQ}{\left(\frac{R}{2}\right)^2}$$

$$= \hat{i}\,\frac{4}{9}\,\frac{kQ}{R^2} - \hat{i}\,4\,\frac{kQ}{R^2}$$

$$= -\hat{i}\,\frac{32}{9}\,\frac{kQ}{R^2}$$

$$E_{tot} = \frac{32}{9}\,\frac{kQ}{R^2}$$