Midterm Exam No. 02 (Spring 2015) PHYS 420: Electricity and Magnetism II

Date: 2015 Mar 18

1. (20 points.) Given the vector differential equation

$$\nabla\phi(\mathbf{r}) = \frac{\mathbf{r} \times (\mathbf{a} \times \mathbf{r})}{r^3} \tag{1}$$

find $\phi(\mathbf{r})$. Here the vector **a** is uniform (constant) with respect to **r**.

2. (20 points.) Calculate the dipole moment

$$\mathbf{d} = \int d^3 r \, \mathbf{r} \, \rho(\mathbf{r}),\tag{2}$$

for the charge density

$$\rho(\mathbf{r}) = (\text{not displayed in preview mode}).$$
(3)

3. (20 points.) A steady current *I* flowing through an infinitely thin wire along the *x*-axis is described by the current density

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{x}} I \delta(z) \delta(y). \tag{4}$$

When the wire is along the y-axis it is described by the current density

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{y}} I\delta(z)\delta(x). \tag{5}$$

The above current densities satisfy

$$\int_{S} d\mathbf{a} \cdot \mathbf{J} = I,\tag{6}$$

where the integration is over an open surface S that crosses the wire once. Write the current density for a wire making an angle θ with respect to the x-axis and in the x-y plane.

Hint: Verify that the current density satisfies Eq. (6) for both the x-z plane and the y-z plane.