Midterm Exam No. 03 (Spring 2015)

PHYS 420: Electricity and Magnetism II

Date: 2015 Apr 15

1. (**20 points.**) This question is not directly related to what we covered in class, but you will, hopefully, appreciate the content. Kindly watch the (54 minute) video at

   `http://www.uctv.tv/shows/The-World-as-a-Hologram-11140`,

   which contains a lecture titled "The World as a Hologram" by Raphael Bousso. Summarize the lecture in about ten sentences using the following concepts: Heisenberg uncertainty principle, blackhole horizon, blackhole entropy, locality, and the holographic principle.

2. (**20 points.**) (Based on Griffiths 4th ed. problem 5.14.)
   A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, if the current is uniformly distributed over the outside surface of the wire.

3. (**20 points.**) (Based on Griffiths 4th ed. problem 5.45.)
   Consider the motion of a particle with mass m and electric charge q_e in the field of a (hypothetical) stationary magnetic monopole q_m held fixed at the origin:

   $$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}\hat{\mathbf{r}}. \tag{1}$$

   (a) Using

   $$\mathbf{F} = q_e \mathbf{v} \times \mathbf{B} \tag{2}$$

   derive the equation of motion

   $$\frac{d\mathbf{v}}{dt} = \mathbf{v} \times \mathbf{r}\,\frac{\mu_0}{4\pi}\frac{q_e q_m}{r^3}\frac{1}{m}, \tag{3}$$

   where $\mathbf{v}$ is velocity of charge q_e.

   (b) Show that the speed $v = |\mathbf{v}|$ is a constant of motion.
   Hint: $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$.

4. (**20 points.**) Is it correct to conclude that

   $$\boldsymbol{\nabla} \cdot (\mathbf{r} \times \mathbf{A}) = -\mathbf{r} \cdot (\boldsymbol{\nabla} \times \mathbf{A}), \tag{4}$$

   where $\mathbf{A}$ is a vector dependent on $\mathbf{r}$? Explain your reasoning.

5. (**20 points.**) The force $d\mathbf{F}$ on an infinitely small line element $d\mathbf{l}$ of wire, carrying steady current I, placed in a (uniform) magnetic field $\mathbf{B}$, is

$$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}. \tag{5}$$

Consider a wire segment of arbitrary shape (in the shape of a curve C) with one end at the origin and the other end at the tip of vector $\mathbf{L}$ when its tail is at the origin.

(a) Complete the line integral (for the case of uniform $\mathbf{B}$)

$$\mathbf{F} = I \int_{\mathbf{0}\,(\text{path } C)}^{\mathbf{L}} d\mathbf{l} \times \mathbf{B} \tag{6}$$

to determine the total force on the wire segment.
Hint: Write $d\mathbf{l} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$ and complete the integrals on x, y, and z.

(b) What is the total force on a loop of wire (of arbitrary shape and carrying steady current I) when it is placed in a uniform magnetic field?