Cylindrical Green's function

Last updated by K. V. Shajesh on January 26, 2015

The cylindrical Green's function satisfies

$$\Big[-\frac{1}{\rho}\frac{\partial}{\partial\rho}\varepsilon(\rho)\rho\frac{\partial}{\partial\rho} + \varepsilon(\rho)\frac{m^2}{\rho^2} + \varepsilon(\rho)k_z^2 \Big] g_m(\rho,\rho';k_z) = \frac{\delta(\rho-\rho')}{\rho}. \tag{1}$$

A dielectric cylinder described by

$$\varepsilon(\rho) = \begin{cases} \varepsilon_2 & \text{for} \quad \rho < a, \\ \varepsilon_1 & \text{for} \quad a < \rho, \end{cases} \tag{2}$$

leads to the solution

$\underline{\rho' < a}$:

$$g_m(\rho,\rho';k_z) = \begin{cases} \dfrac{1}{\varepsilon_2} I_m(k_z\rho_<)K_m(k_z\rho_>) - \dfrac{1}{\varepsilon_2} I_m(k_z\rho)I_m(k_z\rho')\dfrac{K_aK_a'}{\Delta}, & \rho,\rho' < a, \\ \dfrac{1}{\varepsilon_1} I_m(k_z\rho_<)K_m(k_z\rho_>) - \dfrac{1}{\varepsilon_1} K_m(k_z\rho)I_m(k_z\rho')\dfrac{I_a'K_a}{\Delta}, & \rho' < a < \rho. \end{cases} \tag{3}$$

$\underline{a < \rho'}$:

$$g_m(\rho,\rho';k_z) = \begin{cases} \dfrac{1}{\varepsilon_2} I_m(k_z\rho_<)K_m(k_z\rho_>) - \dfrac{1}{\varepsilon_2} I_m(k_z\rho)K_m(k_z\rho')\dfrac{I_aK_a'}{\Delta}, & \rho < a < \rho', \\ \dfrac{1}{\varepsilon_1} I_m(k_z\rho_<)K_m(k_z\rho_>) - \dfrac{1}{\varepsilon_1} K_m(k_z\rho)K_m(k_z\rho')\dfrac{I_aI_a'}{\Delta}, & a < \rho,\rho', \end{cases} \tag{4}$$

where we used the definitions

$$\frac{1}{\Delta} = \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 I_a K_a' - \varepsilon_2 K_a I_a')}, \qquad I_a \equiv I_m(k_z a), \quad K_a \equiv K_m(k_z a). \tag{5}$$

In the perfect conductor limit we have

<u>Inside the cylinder</u>

$$g_m(\rho,\rho';k_z) = \frac{1}{\varepsilon_2} I_m(k_z\rho_<)K_m(k_z\rho_>) - \frac{1}{\varepsilon_2} I_m(k_z\rho)I_m(k_z\rho')\frac{K_a}{I_a}. \tag{6}$$

<u>Outside the cylinder</u>

$$g_m(\rho,\rho';k_z) = \frac{1}{\varepsilon_1} I_m(k_z\rho_<)K_m(k_z\rho_>) - \frac{1}{\varepsilon_1} K_m(k_z\rho)K_m(k_z\rho')\frac{I_a}{K_a}. \tag{7}$$