Homework No. 05 (Spring 2015)

PHYS 420: Electricity and Magnetism II

Due date: Monday, 2015 Mar 16, 4.30pm

1. (**20 points.**) A charged particle initially moving with constant speed v enters a region of magnetic field $\mathbf{B}$ pointing into the page. It is deflected as shown in Fig. 1.

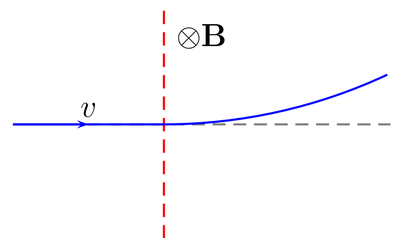

Figure 1: Problem 1

   (a) Is the charge on the particle positive or negative?

   (b) What curve characterizes the path of the deflected particle?

2. (**20 points.**) A steady current I flows through a wire shown in Fig. 2. Find the magnitude and direction of magnetic field at point P.

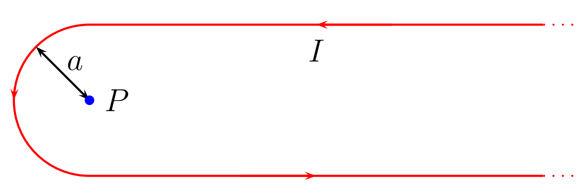

Figure 2: Problem 2

You are given the magnitude of the magnetic field due to an infinite length of wire at distance ρ, and a circular loop of wire of radius R at the center of loop, to be

$$B_{\infty\text{-wire}} = \frac{\mu_0 I}{2\pi\rho} \qquad B_{\text{loop}} = \frac{\mu_0 I}{2R}. \tag{1}$$

3. (**20 points.**) A homogeneous magnetic field $\mathbf{B}$ is characterized by the vector potential

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}. \tag{2}$$

(a) Evaluate $\boldsymbol{\nabla} \times \mathbf{A}$.

(b) Verify that this construction satisfies the radiation gauge by showing that

$$\boldsymbol{\nabla} \cdot \mathbf{A} = 0. \tag{3}$$

(c) Is this construction unique? No. Remember the freedom of gauge transformation,

$$\mathbf{A}' = \mathbf{A} + \boldsymbol{\nabla}\lambda(\mathbf{r}, t), \tag{4}$$

where $\lambda(\mathbf{r}, t)$ is an arbitrary function. Show that for any given vector potential $\mathbf{A}$ there exists λ that satisfies

$$\boldsymbol{\nabla} \cdot \mathbf{A} = -\nabla^2\lambda \tag{5}$$

that leads to the construction of $\mathbf{A}'$ satisfying the radiation gauge.

(d) Let us consider the special case when $\mathbf{B} = B\,\hat{\mathbf{z}}$.

i. Show that, for this case,

$$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r} = -\frac{1}{2}By\,\hat{\mathbf{i}} + \frac{1}{2}Bx\,\hat{\mathbf{j}} = \frac{1}{2}Br\,\hat{\boldsymbol{\phi}}. \tag{6}$$

Visualize $\mathbf{A}$ diagramatically.

ii. Show that

$$\mathbf{A} = 0\,\hat{\mathbf{i}} + Bx\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{z}} \tag{7}$$

is a satisfactory vector potential for homogeneous magnetic field. Visualize $\mathbf{A}$ diagramatically. But, show that this construction does not satisfy the radiation gauge. Using

$$\lambda(\mathbf{r}, t) = \frac{1}{2}Bxy \tag{8}$$

construct $\mathbf{A}'$ that satisfies the radiation gauge.

iii. Show that

$$\mathbf{A} = -By\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{z}} \tag{9}$$

is also a satisfactory vector potential for homogeneous magnetic field. Visualize $\mathbf{A}$ diagramatically. Choose a suitable $\lambda(\mathbf{r}, t)$ to construct $\mathbf{A}'$ that satisfies the radiation gauge.

iv. Chose an arbitray $\lambda(\mathbf{r}, t)$, of your choice, to construct another satisfactory vector potential for homogeneous magnetic field.

4. (**20 points.**) Is the relation

$$(\boldsymbol{\mu} \cdot \boldsymbol{\nabla})\frac{\mathbf{r}}{r^3} = \boldsymbol{\nabla}\left(\frac{\boldsymbol{\mu} \cdot \mathbf{r}}{r^3}\right) \tag{10}$$

correct? ($\boldsymbol{\mu}$ is a position independent vector.)

(a) If yes, prove the relation.

(b) If not, disprove the relation.