Homework No. 06 (Spring 2015)

PHYS 420: Electricity and Magnetism II

Due date: Monday, 2015 Mar 30, 4.30pm

1. (**30 points.**) Motion of a charged particle of mass m and charge q in a uniform magnetic field $\mathbf{B}$ is governed by

$$m\frac{d\mathbf{v}}{dt} = q\,\mathbf{v}\times\mathbf{B}. \tag{1}$$

Choose $\mathbf{B}$ along the z-axis and solve this vector differential equation to determine the position $\mathbf{x}(t)$ and velocity $\mathbf{v}(t)$ of the particle as a function of time, for initial conditions

$$\mathbf{x}(0) = 0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}, \tag{2a}$$
$$\mathbf{v}(0) = 0\,\hat{\mathbf{i}} + v_0\,\hat{\mathbf{j}} + 0\,\hat{\mathbf{k}}. \tag{2b}$$

Verify that the solution describes a circle with center at position $R\,\hat{\mathbf{i}}$.

2. (**20 points.**) The vector potential for a straight wire of infinite extent carrying a steady current I is

$$\mathbf{A}(\mathbf{r}) = \hat{\mathbf{z}}\,\frac{\mu_0 I}{2\pi}\ln\frac{2L}{\rho}, \tag{3}$$

with $L\to\infty$ understood in the equation. The magnetic field around the wire is given by

$$\mathbf{B}(\mathbf{r}) = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi\rho}. \tag{4}$$

   (a) I leave the derivation of the above vector potential as an optional exercise, with bonus points worth **50 points** that could be used towards another homework.

   (b) Using an appropriate diagram describe the above vector potential and the magnetic field.

   (c) Evaluate $\boldsymbol{\nabla}\times\mathbf{A}$.