

Homework No. 07 (Spring 2015)

PHYS 420: Electricity and Magnetism II

Due date: Monday, 2015 Apr 6, 4.30pm

1. **(40 points.)** (Based on Problem 5.58, Griffiths 4th edition.) A circular loop of wire carries a charge q . It rotates with angular velocity $\boldsymbol{\omega}$ about its axis, say z -axis.

(a) Show that the current density generated by this motion is given by

$$\mathbf{J}(\mathbf{r}) = \frac{q}{2\pi a} \boldsymbol{\omega} \times \mathbf{r} \delta(\rho - a) \delta(z - 0). \quad (1)$$

Hint: Use $\mathbf{J}(\mathbf{r}) = \rho(\mathbf{r})\mathbf{v}$, and $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ for circular motion.

(b) Using

$$\mathbf{m} = \frac{1}{2} \int d^3r \mathbf{r} \times \mathbf{J}(\mathbf{r}). \quad (2)$$

determine the magnetic dipole moment of this loop to be

$$\mathbf{m} = \frac{qa^2}{2} \boldsymbol{\omega}. \quad (3)$$

(c) Calculate the angular momentum of the rotating loop to be

$$\mathbf{L} = ma^2 \boldsymbol{\omega}, \quad (4)$$

where m is the mass of the loop.

(d) What is the gyromagnetic ratio g of the rotating loop, which is defined by the relation $\mathbf{m} = g\mathbf{L}$.

2. **(20 points.)** A charged spherical shell carries a charge q . It rotates with angular velocity $\boldsymbol{\omega}$ about a diameter, say z -axis.

(a) Show that the current density generated by this motion is given by

$$\mathbf{J}(\mathbf{r}) = \frac{q}{4\pi a^2} \boldsymbol{\omega} \times \mathbf{r} \delta(r - a). \quad (5)$$

Hint: Use $\mathbf{J}(\mathbf{r}) = \rho(\mathbf{r})\mathbf{v}$ and $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ for circular motion.

(b) Using

$$\mathbf{m} = \frac{1}{2} \int d^3r \mathbf{r} \times \mathbf{J}(\mathbf{r}). \quad (6)$$

determine the magnetic dipole moment of the rotating sphere to be

$$\mathbf{m} = \frac{qa^2}{3} \boldsymbol{\omega}. \quad (7)$$

3. **(30 points.)** A steady current I flows down a long cylindrical wire of radius a . The current density in the wire is described by, $n > 0$,

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{z}} \frac{I}{2\pi a^2} (n+2) \left(\frac{\rho}{a}\right)^n \theta(a-\rho). \quad (8)$$

- (a) Show that, indeed,

$$\int_S d\mathbf{S} \cdot \mathbf{J}(\mathbf{r}) = I. \quad (9)$$

- (b) Using Ampere's law show that the magnetic field inside and outside the cylinder is given by

$$\mathbf{B}(\mathbf{r}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{2I}{\rho} \left(\frac{\rho}{a}\right)^{n+2} \hat{\phi} & \rho < a, \\ \frac{\mu_0}{4\pi} \frac{2I}{\rho} \hat{\phi} & \rho > a. \end{cases} \quad (10)$$

- (c) Plot the magnitude of the magnetic field as a function of ρ .