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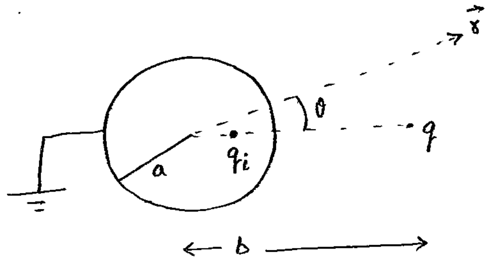
Charge inside a conducting sphere – method of images

① Consider a point charge q outside a perfectly conducting sphere of radius a



② Electric field is zero inside a conductor, and can have only components normal to the surface. Any constant electric potential will achieve this. A grounded conductor means that this constant is chosen to be zero. Thus, we have

$$\phi(\vec{a}) = 0 \quad \text{(grounded conductor)}.$$

③ Using symmetry arguments we conclude

(i) image charge is on the radial line.

(ii) image charge is opposite in sign.

(4)
$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{1}{|\vec{r}-\vec{b}|} - \frac{q_i}{4\pi\epsilon_0}\frac{1}{|\vec{r}-\vec{r}_i|}$$

$$\phi(\vec{a}) = 0$$

$$\Rightarrow \quad \frac{q}{|\vec{a}-\vec{b}|} = \frac{q_i}{|\vec{a}-\vec{r}_i|}$$

$$\frac{q}{\sqrt{a^2+b^2-2ab\cos\theta}} = \frac{q_i}{\sqrt{a^2+r_i^2-2ar_i\cos\theta}}$$

$$q^2(a^2+r_i^2) - q_i^2(a^2+b^2) - 2a(q^2 r_i - q_i^2 b)\cos\theta = 0$$

(5) Since the above equation is true for all angles θ we have

$$q^2(a^2+r_i^2) = q_i^2(a^2+b^2) \qquad \text{— (i)}$$

and

$$\frac{r_i}{b} = \frac{q_i^2}{q^2} \qquad \text{— (ii)}$$

⑥ Together it leads to the quadratic equation

$$\left(\frac{r_i}{a}\right)^2 - \frac{r_i}{a}\left(\frac{a}{b} + \frac{b}{a}\right) + 1 = 0$$

which has solutions

$$\frac{r_i}{a} = \frac{1}{2}\left[\left(\frac{a}{b} + \frac{b}{a}\right) \pm \left(\frac{a}{b} - \frac{b}{a}\right)\right]$$

$$= \frac{a}{b} \quad \text{or} \quad \frac{b}{a}.$$

⑦ Thus, two solutions are

$$\left\{\begin{array}{l} r_i = a\,\frac{a}{b} \\ q_i = q\,\frac{a}{b} \end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{l} r_i = b \\ q_i = q \end{array}\right\}$$

trivial.

Note that $\frac{a}{b} < 1$. Thus, the image charge is always inside the sphere.