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(1) Magnetostatics is governed by the equations

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} \quad \Rightarrow \quad \vec{\nabla} \cdot \vec{j} = 0$$

(2) For an infinitely long wire of negligible thickness we have

$$\vec{j}(\vec{r}) = \hat{z}\, I\, \delta(x)\, \delta(y)$$

(3) The magnetic field is determined using

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int d^3r'\, \vec{j}(\vec{r}\,') \times \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3}$$

$$= \frac{\mu_0}{4\pi} \int_{-\infty}^{+\infty} dz' \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dy'\, \hat{z}\, I\, \delta(x')\, \delta(y') \times \frac{(x-x')\,\hat{i} + (y-y')\,\hat{j} + (z-z')\,\hat{z}}{\left[\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}\right]^3}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} dz' \frac{x\,\hat{j} - y\,\hat{i}}{\left[\sqrt{x^2 + y^2 + (z-z')^2}\right]^3}$$

(4) Let

$$x^2 + y^2 = \rho^2 \qquad \hat{\rho} = \frac{x\,\hat{i} + y\,\hat{j}}{\rho}$$

$$\tan\phi = \frac{y}{x} \qquad \hat{\phi} = \frac{x\,\hat{j} - y\,\hat{i}}{\rho}$$

Thus,

$$\vec{B}(\vec{r}) = \hat{\phi}\, \frac{\mu_0 I}{4\pi}\, \rho \int_{-\infty}^{+\infty} dz' \frac{1}{\left[\rho^2 + (z-z')^2\right]^{\frac{3}{2}}}$$

5 Substituting

$$z' - z = \varrho \tan\theta \qquad dz' = \varrho \sec^2\theta \, d\theta .$$

$$\vec{B}(\vec{r}) = \hat{\phi} \frac{\mu_0 I}{4\pi} \varrho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\varrho \sec^2\theta \, d\theta}{\varrho^3 \sec^3\theta}$$

$$= \hat{\phi} \frac{\mu_0 I}{4\pi\varrho} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, d\theta .$$

$$= \hat{\phi} \frac{\mu_0 I}{4\pi\varrho} \underbrace{\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) \right]}_{=2}$$

$$= \hat{\phi} \frac{\mu_0 I}{2\pi\varrho}$$