Date: Mar 20, 2015

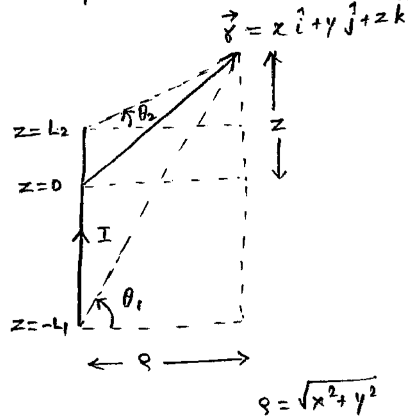
(1) Magnetostatics is governed by

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A}$$

gauge choice $\vec{\nabla}\cdot\vec{A} = 0$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} \quad \Rightarrow \quad \vec{\nabla}\cdot\vec{j} = 0$$

(2) Consider a current carrying wire of finite length.



$$\vec{j}(\vec{r}) = \hat{z}\, I\, \delta(x)\delta(y)\; \theta(-L_1 < z < L_2)$$

(3)

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int d^3r'\; \vec{j}(\vec{r}') \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$= \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} dz' \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dy'\; \hat{z}\, I\, \delta(x')\delta(y')\; \theta(-L_1 < z' < L_2) \times \frac{(x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\int_{-L_1}^{L_2} dz'\; \frac{x\hat{j} - y\hat{i}}{\left[x^2+y^2+(z-z')^2\right]^{\frac{3}{2}}}$$

$$= \hat{\phi}\,\frac{\mu_0 I}{4\pi}\,\rho\int_{-L_1}^{L_2} dz'\; \frac{1}{\left[\rho^2 + (z'-z)^2\right]^{\frac{3}{2}}}$$

$z' - z = \rho\tan\theta$.
$dz' = \rho\sec^2\theta\, d\theta$.

$$= \hat{\phi}\,\frac{\mu_0 I}{4\pi}\,\rho\int_{\tan^{-1}\frac{-L_1-z}{\rho}}^{\tan^{-1}\frac{L_2-z}{\rho}} \underbrace{\frac{\rho\sec^2\theta\, d\theta}{\rho^3\sec^3\theta}}_{\frac{1}{\rho}\cos\theta\, d\theta}$$

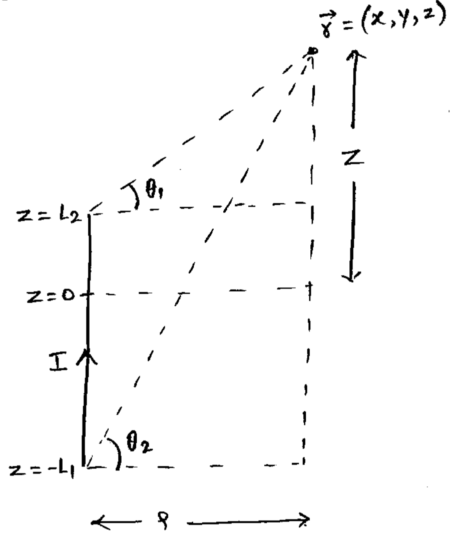
$$= \hat{\phi}\,\frac{\mu_0 I}{4\pi\rho}\,\sin\theta\,\Bigg|_{\theta=\tan^{-1}\frac{-L_1-z}{\rho}}^{\theta=\tan^{-1}\frac{L_2-z}{\rho}}$$

4 $$\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

$$\vec{B}(\vec{r}) = \hat{\phi}\,\frac{\mu_0 I}{4\pi\rho}\left[\frac{\frac{L_2-z}{\rho}}{\sqrt{1+\left(\frac{L_2-z}{\rho}\right)^2}} - \frac{\left(\frac{-L_1-z}{\rho}\right)}{\sqrt{1+\left(\frac{-L_1-z}{\rho}\right)^2}}\right]$$

$$= \hat{\phi}\,\frac{\mu_0 I}{4\pi\rho}\left[\frac{z-(-L_1)}{\sqrt{\rho^2+(z+L_1)^2}} - \frac{(z-L_2)}{\sqrt{\rho^2+(z-L_2)^2}}\right]$$

$$= \hat{\phi}\,\frac{\mu_0 I}{4\pi\rho}\left[\sin\theta_1 - \sin\theta_2\right]$$



5 For $L_1 \to \infty$ $\left(\theta_1 \to +\frac{\pi}{2}\right)$

$L_2 \to \infty$ $\left(\theta_2 \to -\frac{\pi}{2}\right)$

we immediately have

$$\vec{B}(\vec{r}) = \hat{\phi}\,\frac{\mu_0 I}{2\pi\rho}$$