Midterm Exam No. 01 (Spring 2015)

PHYS 520B: Electromagnetic Theory

Date: 2014 Feb 11

1. (**20 points.**) The gradient operator in cylinderical coordinates (ρ, ϕ, z) is

$$\boldsymbol{\nabla} = \hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \hat{\boldsymbol{\phi}} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial}{\partial z}. \tag{1}$$

The electric field of an infinitely long rod of negligible thickness is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{\rho} \hat{\boldsymbol{\rho}}, \tag{2}$$

where λ is the charge per unit length on the rod. Evaluate

$$\boldsymbol{\nabla} \cdot \mathbf{E}. \tag{3}$$

Hint: The divergence of electric field at a point in space is a measure of the charge density at that point. It satisfies the Gauss's law.

2. (**20 points.**) The modified Bessel function of zeroth order has the following asymptotic form near $t = 0$,

$$K_0(t) \sim \ln \frac{2}{t} - \gamma, \quad t \ll 1, \tag{4}$$

where $\gamma = 0.1159\ldots$ is the Euler's constant. Evaluate the limit

$$\lim_{t \to 0} \frac{K_0(at)}{K_0(t)} \tag{5}$$

for positive real a.

3. (**60 points.**) Consider a wire of infinite length and negligible thickness inside a cylindrical cavity with perfectly conducting walls. The cylindrical cavity in cross section forms a circle of radius a. The Green's function inside such a perfectly conducting cylinder is

$$\begin{aligned} G(\mathbf{r}, \mathbf{r}') = {} & \frac{1}{\varepsilon_0} \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(z-z')} \sum_{m=-\infty}^{\infty} \frac{1}{2\pi} e^{im(\phi - \phi')} \\ & \times \left[I_m(k\rho_<) K_m(k\rho_>) - \frac{K_m(ka)}{I_m(ka)} I_m(k\rho) I_m(k\rho') \right]. \end{aligned} \tag{6}$$

The charge density of wire in cylindrical coordinates is

$$\rho(\mathbf{r}) = \lambda \frac{\delta(\rho - \rho_0)}{\rho} \delta(\phi - \phi_0), \tag{7}$$

where λ is the charge per length and ρ_0, ϕ_0 specify the position of the wire. For the case when the wire is on the axis of the cylinder we have $\rho_0 = 0$ and $\phi_0 = 0$. For this elementary case evaluate the electric potential using the relation

$$\phi(\mathbf{r}) = \int d^3r' G(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}'). \tag{8}$$

Hint: All the integrals and sum can be completed for this case.