Homework No. 02 (Spring 2015)

PHYS 520B: Electromagnetic Theory

Due date: Monday, 2015 Feb 9, 4.30pm

1. (**50 points.**) Consider a point charge q placed a radial distance $\rho_0 > a$ away from the axis of a perfectly conducting cylinder. Here a is the radius of the cylinder.

   (a) Using the connection between the electric potential and Green's function,
   $$\phi(\mathbf{r}) = q\, G(\mathbf{r}, \mathbf{r}_0), \tag{1}$$
   and the Green function for a perfectly conducting cylinder, derived in class, determine the electric potential to be
   $$\phi(\mathbf{r}) = \frac{q}{\varepsilon_0} \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikz} \sum_{-\infty}^{\infty} \frac{1}{2\pi} e^{im\phi} \left[I_m(k\rho_<)K_m(k\rho_>) - \frac{I_m(ka)}{K_m(ka)} K_m(k\rho)K_m(k\rho_0) \right], \tag{2}$$
   where $\mathbf{r} = (\rho, \phi, z)$ and the position of the point charge $\mathbf{r}_0$ is chosen to be $(\rho_0, 0, 0)$.

   (b) Verify that the potential satisfies the boundary condition
   $$\phi(\mathbf{a}) = 0 \tag{3}$$
   on the surface of the conducting cylinder.

   (c) Using the relation $\mathbf{E} = -\boldsymbol{\nabla}\phi$ evaluate the electric field on the surface of the conductor to be
   $$\mathbf{E}(\mathbf{a}) = -\hat{\boldsymbol{\rho}} \frac{q}{\varepsilon_0} \frac{1}{a} \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikz} \sum_{-\infty}^{\infty} \frac{1}{2\pi} e^{im\phi} \frac{K_m(k\rho_0)}{K_m(ka)}. \tag{4}$$
   Note that the electric field is normal to the surface of the cylinder.

   (d) Using Gauss's theorem we can argue that the induced charge on the surface of a conductor is given using
   $$\sigma(\phi, z) = \varepsilon_0 \hat{\mathbf{n}} \cdot \mathbf{E}\Big|_{\text{surface}}, \tag{5}$$
   where $\hat{\mathbf{n}}$ is normal to the surface of conductor. Thus, determine the induced charge density $\sigma(\phi, z)$ on the surface of the cylinder.

   (e) By integrating over the surface of the cylinder determine the total induced charge on the cylinder. Thus, find out if its magnitude is less than, equal to, or greater than, the charge q.