Midterm Exam No. 02 (Spring 2015)

PHYS 530A: Quantum Mechanics II

Date: 2015 Mar 23

1. (**20 points.**) The probabilities in a series setup of Stern-Gerlach experiment, for spin-$\frac{1}{2}$, is given by

$$p([A=a'] \to [B=b'] \to [C=c']) = \mathrm{tr}\left(\frac{1+a'A}{2}\right)\left(\frac{1+b'B}{2}\right)\left(\frac{1+c'C}{2}\right), \tag{1}$$

where $[A=a']$ denotes the selection of the beam corresponding to the eigenvalue a'.

   (a) Find the following probabilities:
      i. $p([\sigma_x=+1] \to [\sigma_x=+1])$
      ii. $p([\sigma_x=+1] \to [\sigma_y=+1] \to [\sigma_x=+1])$
   (b) Does the measurement of σ_y *completely* wipe out the prior knowledge of the measurement of σ_x? If yes, why? If no, why not?

2. (**20 points.**) A unitary matrix is defined by

$$U^\dagger U = 1, \tag{2}$$

where † stands for transpose and complex conjugation. Show that

$$U = \frac{1+iA}{1-iA} \tag{3}$$

is unitary if A is Hermitian, that is $A^\dagger = A$.

3. (**20 points.**) If $\boldsymbol{\sigma}$ is the vector constructed out of the Pauli matrices and $\mathbf{a}$ is a (numerical) vector, evaluate

$$\mathrm{tr}\cos(\boldsymbol{\sigma}\cdot\mathbf{a}). \tag{4}$$

4. (**20 points.**) Evaluate

$$[(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b})]\,(\boldsymbol{\sigma}\cdot\mathbf{c}). \tag{5}$$

Then evaluate

$$(\boldsymbol{\sigma}\cdot\mathbf{a})\,[(\boldsymbol{\sigma}\cdot\mathbf{b})(\boldsymbol{\sigma}\cdot\mathbf{c})]\,. \tag{6}$$

Are they equal?

5. (**20 points.**) The Pauli matrix

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{7}$$

is written in the eigenbasis of σ_z. Write σ_x in the eigenbasis of σ_y.