Final Exam (Spring 2015)

PHYS 530A: Quantum Mechanics II

Due date: Thursday, 2015 May 14

1. (**50 points.**) Homework 09.

2. (**20 points.**) Consider the Hamiltonian

$$H = \frac{p^2}{2\mu} + V(\mathbf{r}). \tag{1}$$

Under what conditions is $r = |\mathbf{r}|$ a conserved quantity? Describe the path of motion when r is a conserved quantity.

3. (**30 points.**) Using commutation relations between $\mathbf{r}$, $\mathbf{p}$, and $\mathbf{L}$, verify the relation

$$\mathbf{p} \times \mathbf{L} \cdot \mathbf{p} = 2i\hbar\, p^2. \tag{2}$$

Thus, verify that either of the equalities for

$$\mathbf{M} = -\frac{1}{2}\left(\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}\right) = -\mathbf{p} \times \mathbf{L} + i\hbar\mathbf{p} = \mathbf{L} \times \mathbf{p} - i\hbar\mathbf{p} \tag{3}$$

leads to

$$M^2 = (L^2 + \hbar^2)p^2. \tag{4}$$

Comment: This ensures that either of the expressions for the Axial vector

$$\mathbf{A} = \hat{\mathbf{r}} - \frac{1}{\mu Z e^2}\frac{1}{2}\left(\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}\right) \tag{5a}$$

$$= \hat{\mathbf{r}} - \frac{1}{\mu Z e^2}\mathbf{p} \times \mathbf{L} + \frac{i\hbar}{\mu Z e^2}\mathbf{p} \tag{5b}$$

$$= \hat{\mathbf{r}} + \frac{1}{\mu Z e^2}\mathbf{L} \times \mathbf{p} - \frac{i\hbar}{\mu Z e^2}\mathbf{p} \tag{5c}$$

leads to

$$A^2 = 1 + \frac{2(L^2 + \hbar^2)H}{\mu Z e^2}. \tag{6}$$