Homework No. 03 (Spring 2015)

PHYS 530A: Quantum Mechanics II

Due date: Friday, 2015 Feb 27, 4.30pm

1. (**20 points.**) (Ref: Milton's notes.) The energy of a charge e moving with velocity $\mathbf{v}$ in an external electromagnetic field is

$$E = e\phi - \frac{e}{c}\mathbf{v}\cdot\mathbf{A}, \tag{1}$$

where ϕ is the scalar potential and $\mathbf{A}$ is the vector potential. The relation between $\mathbf{A}$ and the magnetic field $\mathbf{H}$ is

$$\mathbf{H} = \boldsymbol{\nabla}\times\mathbf{A}. \tag{2}$$

For a constant (homogenous in space) magnetic field $\mathbf{H}$, verify that

$$\mathbf{A} = \frac{1}{2}\mathbf{H}\times\mathbf{r} \tag{3}$$

is a possible vector potential. Then, by looking at the energy, identify the magnetic moment $\boldsymbol{\mu}$ of the moving charge.

2. (**20 points.**) (Ref: Milton's notes.) Consider an atom entering a Stern-Gerlach apparatus. Deflection upward begins as soon as the atom enters the inhomogeneous field. By the time the atom leaves the field, it has been deflected upward by a net amount Δz. Compute Δz for

$$\mu_z = 10^{-27}\frac{\text{J}}{\text{G}}, \quad \frac{\partial H_z}{\partial z} = 10^6\frac{\text{G}}{\text{m}}, \quad l = 10\,\text{cm}, \quad T = \frac{mv_x^2}{k} = 10^3\,\text{K}. \tag{4}$$

3. (**20 points.**) (Ref: Milton's notes.) Using the notation for the probability for a measurement in the Stern-Gerlach experiment, introduced in the class, show that

$$p\big([+;\theta_1,\phi_1] \to [-;\theta_2,\phi_2]\big) = \frac{1-\cos\Theta}{2}, \tag{5}$$

where

$$\cos\Theta = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2). \tag{6}$$

4. (**20 points.**) (Ref: Milton's notes.) Show that

$$p\big([+;0,0] \to [+;\pi,0]\big) = 0. \tag{7}$$

Further, show that

$$p\big([+;0,0] \to [\pm;\theta,\phi] \to [+;\pi,0]\big) = 0, \tag{8}$$

which is a statement of destructive interference. Compare this with the probability for

$$p\big([+;0,0] \to [+;\theta,\phi] \to [+;\pi,0]\big) \tag{9}$$

and

$$p\big([+;0,0] \to [-;\theta,\phi] \to [+;\pi,0]\big). \tag{10}$$

5. (**20 points.**) Show that

$$p\big([+;0,0] \to [-;\pi,0]\big) = 1. \tag{11}$$

Further, show that

$$p\big([+;0,0] \to [\pm;\theta,\phi] \to [-;\pi,0]\big) = 1, \tag{12}$$

which is a statement of constructive interference. Compare this with the probability for

$$p\big([+;0,0] \to [+;\theta,\phi] \to [-;\pi,0]\big) \tag{13}$$

and

$$p\big([+;0,0] \to [-;\theta,\phi] \to [-;\pi,0]\big). \tag{14}$$